

FHSST Authors

**The Free High School Science Texts:  
Textbooks for High School Students  
Studying the Sciences  
Physics  
Grades 10 - 12**

**Version 0  
November 9, 2008**

Copyright 2007 "Free High School Science Texts"

Permission **is** granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

**STOP!!!!**

Did you notice the **FREEDOMS** we've granted you?

Our copyright license is **different!** It grants freedoms rather than just imposing restrictions like all those other textbooks you probably own or use.

- We know people copy textbooks illegally but we would LOVE it if you copied our's - go ahead copy to your hearts content, **legally!**
- Publishers' revenue is generated by controlling the market, we don't want any money, go ahead, distribute our books far and wide - we DARE you!
- Ever wanted to change your textbook? Of course you have! Go ahead, change ours, make your own version, get your friends together, rip it apart and put it back together the way you like it. That's what we really want!
- Copy, modify, adapt, enhance, share, critique, adore, and contextualise. Do it all, do it with your colleagues, your friends, or alone but get involved! Together we can overcome the challenges our complex and diverse country presents.
- So what is the catch? The only thing you can't do is take this book, make a few changes and then tell others that they can't do the same with your changes. It's share and share-alike and we know you'll agree that is only fair.
- These books were written by volunteers who want to help support education, who want the facts to be freely available for teachers to copy, adapt and re-use. Thousands of hours went into making them and they are a gift to everyone in the education community.

---

## FHSST Core Team

Mark Horner ; Samuel Halliday ; Sarah Blyth ; Rory Adams ; Spencer Wheaton

## FHSST Editors

Jayne Padayachee ; Joanne Boulle ; Diana Mulcahy ; Annette Nell ; René Toerien ; Donovan Whitfield

## FHSST Contributors

Rory Adams ; Prashant Arora ; Richard Baxter ; Dr. Sarah Blyth ; Sebastian Bodenstein ; Graeme Broster ; Richard Case ; Brett Cocks ; Tim Crombie ; Dr. Anne Dabrowski ; Laura Daniels ; Sean Dobbs ; Fernando Durrell ; Dr. Dan Dwyer ; Frans van Eeden ; Giovanni Franzoni ; Ingrid von Glehn ; Tamara von Glehn ; Lindsay Glesener ; Dr. Vanessa Godfrey ; Dr. Johan Gonzalez ; Hemant Gopal ; Umeshree Govender ; Heather Gray ; Lynn Greeff ; Dr. Tom Gutierrez ; Brooke Haag ; Kate Hadley ; Dr. Sam Halliday ; Asheena Hanuman ; Neil Hart ; Nicholas Hatcher ; Dr. Mark Horner ; Robert Hovden ; Mfandaidza Hove ; Jennifer Hsieh ; Clare Johnson ; Luke Jordan ; Tana Joseph ; Dr. Jennifer Klay ; Lara Kruger ; Sihle Kubheka ; Andrew Kubik ; Dr. Marco van Leeuwen ; Dr. Anton Machacek ; Dr. Komal Maheshwari ; Kosma von Maltitz ; Nicole Masureik ; John Mathew ; JoEllen McBride ; Nikolai Meures ; Riana Meyer ; Jenny Miller ; Abdul Mirza ; Asogan Moodaly ; Jothi Moodley ; Nolene Naidu ; Tyrone Negus ; Thomas O'Donnell ; Dr. Markus Oldenburg ; Dr. Jaynie Padayachee ; Nicolette Pekeur ; Sirika Pillay ; Jacques Plaut ; Andrea Prinsloo ; Joseph Raimondo ; Sanya Rajani ; Prof. Sergey Rakityansky ; Alastair Ramlakan ; Razvan Remsing ; Max Richter ; Sean Riddle ; Evan Robinson ; Dr. Andrew Rose ; Bianca Ruddy ; Katie Russell ; Duncan Scott ; Helen Seals ; Ian Sherratt ; Roger Sieloff ; Bradley Smith ; Greg Solomon ; Mike Stringer ; Shen Tian ; Robert Torregrosa ; Jimmy Tseng ; Helen Waugh ; Dr. Dawn Webber ; Michelle Wen ; Dr. Alexander Wetzler ; Dr. Spencer Wheaton ; Vivian White ; Dr. Gerald Wigger ; Harry Wiggins ; Wendy Williams ; Julie Wilson ; Andrew Wood ; Emma Wormauld ; Sahal Yacoob ; Jean Youssef

Contributors and editors have made a sincere effort to produce an accurate and useful resource. Should you have suggestions, find mistakes or be prepared to donate material for inclusion, please don't hesitate to contact us. We intend to work with all who are willing to help make this a continuously evolving resource!



# Contents

<b>I Introduction</b>	<b>1</b>
<b>1 What is Physics?</b>	<b>3</b>
<b>II Grade 10 - Physics</b>	<b>5</b>
<b>2 Units</b>	<b>9</b>
2.1 Introduction . . . . .	9
2.2 Unit Systems . . . . .	9
2.2.1 SI Units . . . . .	9
2.2.2 The Other Systems of Units . . . . .	10
2.3 Writing Units as Words or Symbols . . . . .	10
2.4 Combinations of SI Base Units . . . . .	12
2.5 Rounding, Scientific Notation and Significant Figures . . . . .	12
2.5.1 Rounding Off . . . . .	12
2.5.2 Error Margins . . . . .	13
2.5.3 Scientific Notation . . . . .	13
2.5.4 Significant Figures . . . . .	15
2.6 Prefixes of Base Units . . . . .	15
2.7 The Importance of Units . . . . .	17
2.8 How to Change Units . . . . .	17
2.8.1 Two other useful conversions . . . . .	19
2.9 A sanity test . . . . .	19
2.10 Summary . . . . .	19
2.11 End of Chapter Exercises . . . . .	21
<b>3 Motion in One Dimension - Grade 10</b>	<b>23</b>
3.1 Introduction . . . . .	23
3.2 Reference Point, Frame of Reference and Position . . . . .	23
3.2.1 Frames of Reference . . . . .	23
3.2.2 Position . . . . .	25
3.3 Displacement and Distance . . . . .	28
3.3.1 Interpreting Direction . . . . .	29
3.3.2 Differences between Distance and Displacement . . . . .	29
3.4 Speed, Average Velocity and Instantaneous Velocity . . . . .	31

3.4.1 Differences between Speed and Velocity . . . . .	35
3.5 Acceleration . . . . .	38
3.6 Description of Motion . . . . .	39
3.6.1 Stationary Object . . . . .	40
3.6.2 Motion at Constant Velocity . . . . .	41
3.6.3 Motion at Constant Acceleration . . . . .	46
3.7 Summary of Graphs . . . . .	48
3.8 Worked Examples . . . . .	49
3.9 Equations of Motion . . . . .	54
3.9.1 Finding the Equations of Motion . . . . .	54
3.10 Applications in the Real-World . . . . .	59
3.11 Summary . . . . .	61
3.12 End of Chapter Exercises: Motion in One Dimension . . . . .	62
<b>4 Gravity and Mechanical Energy - Grade 10</b>	<b>67</b>
4.1 Weight . . . . .	67
4.1.1 Differences between Mass and Weight . . . . .	68
4.2 Acceleration due to Gravity . . . . .	69
4.2.1 Gravitational Fields . . . . .	69
4.2.2 Free fall . . . . .	69
4.3 Potential Energy . . . . .	73
4.4 Kinetic Energy . . . . .	75
4.4.1 Checking units . . . . .	77
4.5 Mechanical Energy . . . . .	78
4.5.1 Conservation of Mechanical Energy . . . . .	78
4.5.2 Using the Law of Conservation of Energy . . . . .	79
4.6 Energy graphs . . . . .	82
4.7 Summary . . . . .	83
4.8 End of Chapter Exercises: Gravity and Mechanical Energy . . . . .	84
<b>5 Transverse Pulses - Grade 10</b>	<b>87</b>
5.1 Introduction . . . . .	87
5.2 What is a <i>medium</i> ? . . . . .	87
5.3 What is a <i>pulse</i> ? . . . . .	87
5.3.1 Pulse Length and Amplitude . . . . .	88
5.3.2 Pulse Speed . . . . .	89
5.4 Graphs of Position and Velocity . . . . .	90
5.4.1 Motion of a Particle of the Medium . . . . .	90
5.4.2 Motion of the Pulse . . . . .	92
5.5 Transmission and Reflection of a Pulse at a Boundary . . . . .	96
5.6 Reflection of a Pulse from Fixed and Free Ends . . . . .	97
5.6.1 Reflection of a Pulse from a Fixed End . . . . .	97

5.6.2	Reflection of a Pulse from a Free End . . . . .	98
5.7	Superposition of Pulses . . . . .	99
5.8	Exercises - Transverse Pulses . . . . .	102
<b>6</b>	<b>Transverse Waves - Grade 10</b>	<b>105</b>
6.1	Introduction . . . . .	105
6.2	What is a <i>transverse wave</i> ? . . . . .	105
6.2.1	Peaks and Troughs . . . . .	106
6.2.2	Amplitude and Wavelength . . . . .	107
6.2.3	Points in Phase . . . . .	109
6.2.4	Period and Frequency . . . . .	110
6.2.5	Speed of a Transverse Wave . . . . .	111
6.3	Graphs of Particle Motion . . . . .	115
6.4	Standing Waves and Boundary Conditions . . . . .	118
6.4.1	Reflection of a Transverse Wave from a Fixed End . . . . .	118
6.4.2	Reflection of a Transverse Wave from a Free End . . . . .	118
6.4.3	Standing Waves . . . . .	118
6.4.4	Nodes and anti-nodes . . . . .	122
6.4.5	Wavelengths of Standing Waves with Fixed and Free Ends . . . . .	122
6.4.6	Superposition and Interference . . . . .	125
6.5	Summary . . . . .	127
6.6	Exercises . . . . .	127
<b>7</b>	<b>Geometrical Optics - Grade 10</b>	<b>129</b>
7.1	Introduction . . . . .	129
7.2	Light Rays . . . . .	129
7.2.1	Shadows . . . . .	132
7.2.2	Ray Diagrams . . . . .	132
7.3	Reflection . . . . .	132
7.3.1	Terminology . . . . .	133
7.3.2	Law of Reflection . . . . .	133
7.3.3	Types of Reflection . . . . .	135
7.4	Refraction . . . . .	137
7.4.1	Refractive Index . . . . .	139
7.4.2	Snell's Law . . . . .	139
7.4.3	Apparent Depth . . . . .	143
7.5	Mirrors . . . . .	146
7.5.1	Image Formation . . . . .	146
7.5.2	Plane Mirrors . . . . .	147
7.5.3	Ray Diagrams . . . . .	148
7.5.4	Spherical Mirrors . . . . .	150
7.5.5	Concave Mirrors . . . . .	150

7.5.6 Convex Mirrors . . . . .	153
7.5.7 Summary of Properties of Mirrors . . . . .	154
7.5.8 Magnification . . . . .	154
7.6 Total Internal Reflection and Fibre Optics . . . . .	156
7.6.1 Total Internal Reflection . . . . .	156
7.6.2 Fibre Optics . . . . .	161
7.7 Summary . . . . .	163
7.8 Exercises . . . . .	164
<b>8 Magnetism - Grade 10</b>	<b>167</b>
8.1 Introduction . . . . .	167
8.2 Magnetic fields . . . . .	167
8.3 Permanent magnets . . . . .	169
8.3.1 The poles of permanent magnets . . . . .	169
8.3.2 Magnetic attraction and repulsion . . . . .	169
8.3.3 Representing magnetic fields . . . . .	170
8.4 The compass and the earth's magnetic field . . . . .	173
8.4.1 The earth's magnetic field . . . . .	175
8.5 Summary . . . . .	175
8.6 End of chapter exercises . . . . .	176
<b>9 Electrostatics - Grade 10</b>	<b>177</b>
9.1 Introduction . . . . .	177
9.2 Two kinds of charge . . . . .	177
9.3 Unit of charge . . . . .	177
9.4 Conservation of charge . . . . .	177
9.5 Force between Charges . . . . .	178
9.6 Conductors and insulators . . . . .	181
9.6.1 The electroscope . . . . .	182
9.7 Attraction between charged and uncharged objects . . . . .	183
9.7.1 Polarisation of Insulators . . . . .	183
9.8 Summary . . . . .	184
9.9 End of chapter exercise . . . . .	184
<b>10 Electric Circuits - Grade 10</b>	<b>187</b>
10.1 Electric Circuits . . . . .	187
10.1.1 Closed circuits . . . . .	187
10.1.2 Representing electric circuits . . . . .	188
10.2 Potential Difference . . . . .	192
10.2.1 Potential Difference . . . . .	192
10.2.2 Potential Difference and Parallel Resistors . . . . .	193
10.2.3 Potential Difference and Series Resistors . . . . .	194
10.2.4 Ohm's Law . . . . .	194

10.2.5 EMF . . . . .	195
10.3 Current . . . . .	198
10.3.1 Flow of Charge . . . . .	198
10.3.2 Current . . . . .	198
10.3.3 Series Circuits . . . . .	199
10.3.4 Parallel Circuits . . . . .	200
10.4 Resistance . . . . .	202
10.4.1 What causes resistance? . . . . .	202
10.4.2 Resistors in electric circuits . . . . .	202
10.5 Instruments to Measure voltage, current and resistance . . . . .	204
10.5.1 Voltmeter . . . . .	204
10.5.2 Ammeter . . . . .	204
10.5.3 Ohmmeter . . . . .	204
10.5.4 Meters Impact on Circuit . . . . .	205
10.6 Exercises - Electric circuits . . . . .	205

### III Grade 11 - Physics 209

<b>11 Vectors</b>	<b>211</b>
11.1 Introduction . . . . .	211
11.2 Scalars and Vectors . . . . .	211
11.3 Notation . . . . .	211
11.3.1 Mathematical Representation . . . . .	212
11.3.2 Graphical Representation . . . . .	212
11.4 Directions . . . . .	212
11.4.1 Relative Directions . . . . .	212
11.4.2 Compass Directions . . . . .	213
11.4.3 Bearing . . . . .	213
11.5 Drawing Vectors . . . . .	214
11.6 Mathematical Properties of Vectors . . . . .	215
11.6.1 Adding Vectors . . . . .	215
11.6.2 Subtracting Vectors . . . . .	217
11.6.3 Scalar Multiplication . . . . .	218
11.7 Techniques of Vector Addition . . . . .	218
11.7.1 Graphical Techniques . . . . .	218
11.7.2 Algebraic Addition and Subtraction of Vectors . . . . .	223
11.8 Components of Vectors . . . . .	228
11.8.1 Vector addition using components . . . . .	231
11.8.2 Summary . . . . .	235
11.8.3 End of chapter exercises: Vectors . . . . .	236
11.8.4 End of chapter exercises: Vectors - Long questions . . . . .	237

<b>12 Force, Momentum and Impulse - Grade 11</b>	<b>239</b>
12.1 Introduction . . . . .	239
12.2 Force . . . . .	239
12.2.1 What is a <i>force</i> ? . . . . .	239
12.2.2 Examples of Forces in Physics . . . . .	240
12.2.3 Systems and External Forces . . . . .	241
12.2.4 Force Diagrams . . . . .	242
12.2.5 Free Body Diagrams . . . . .	243
12.2.6 Finding the Resultant Force . . . . .	244
12.2.7 Exercise . . . . .	246
12.3 Newton's Laws . . . . .	246
12.3.1 Newton's First Law . . . . .	247
12.3.2 Newton's Second Law of Motion . . . . .	249
12.3.3 Exercise . . . . .	261
12.3.4 Newton's Third Law of Motion . . . . .	263
12.3.5 Exercise . . . . .	267
12.3.6 Different types of forces . . . . .	268
12.3.7 Exercise . . . . .	275
12.3.8 Forces in equilibrium . . . . .	276
12.3.9 Exercise . . . . .	279
12.4 Forces between Masses . . . . .	282
12.4.1 Newton's Law of Universal Gravitation . . . . .	282
12.4.2 Comparative Problems . . . . .	284
12.4.3 Exercise . . . . .	286
12.5 Momentum and Impulse . . . . .	287
12.5.1 Vector Nature of Momentum . . . . .	290
12.5.2 Exercise . . . . .	291
12.5.3 Change in Momentum . . . . .	291
12.5.4 Exercise . . . . .	293
12.5.5 Newton's Second Law revisited . . . . .	293
12.5.6 Impulse . . . . .	294
12.5.7 Exercise . . . . .	296
12.5.8 Conservation of Momentum . . . . .	297
12.5.9 Physics in Action: Impulse . . . . .	300
12.5.10 Exercise . . . . .	301
12.6 Torque and Levers . . . . .	302
12.6.1 Torque . . . . .	302
12.6.2 Mechanical Advantage and Levers . . . . .	305
12.6.3 Classes of levers . . . . .	307
12.6.4 Exercise . . . . .	308
12.7 Summary . . . . .	309
12.8 End of Chapter exercises . . . . .	310

<b>13 Geometrical Optics - Grade 11</b>	<b>327</b>
13.1 Introduction . . . . .	327
13.2 Lenses . . . . .	327
13.2.1 Converging Lenses . . . . .	329
13.2.2 Diverging Lenses . . . . .	340
13.2.3 Summary of Image Properties . . . . .	343
13.3 The Human Eye . . . . .	344
13.3.1 Structure of the Eye . . . . .	345
13.3.2 Defects of Vision . . . . .	346
13.4 Gravitational Lenses . . . . .	347
13.5 Telescopes . . . . .	347
13.5.1 Refracting Telescopes . . . . .	347
13.5.2 Reflecting Telescopes . . . . .	348
13.5.3 Southern African Large Telescope . . . . .	348
13.6 Microscopes . . . . .	349
13.7 Summary . . . . .	351
13.8 Exercises . . . . .	352
 <b>14 Longitudinal Waves - Grade 11</b>	<b>355</b>
14.1 Introduction . . . . .	355
14.2 What is a <i>longitudinal wave</i> ? . . . . .	355
14.3 Characteristics of Longitudinal Waves . . . . .	356
14.3.1 Compression and Rarefaction . . . . .	356
14.3.2 Wavelength and Amplitude . . . . .	357
14.3.3 Period and Frequency . . . . .	357
14.3.4 Speed of a Longitudinal Wave . . . . .	358
14.4 Graphs of Particle Position, Displacement, Velocity and Acceleration . . . . .	359
14.5 Sound Waves . . . . .	360
14.6 Seismic Waves . . . . .	361
14.7 Summary - Longitudinal Waves . . . . .	361
14.8 Exercises - Longitudinal Waves . . . . .	362
 <b>15 Sound - Grade 11</b>	<b>363</b>
15.1 Introduction . . . . .	363
15.2 Characteristics of a Sound Wave . . . . .	363
15.2.1 Pitch . . . . .	364
15.2.2 Loudness . . . . .	364
15.2.3 Tone . . . . .	364
15.3 Speed of Sound . . . . .	365
15.4 Physics of the Ear and Hearing . . . . .	365
15.4.1 Intensity of Sound . . . . .	366
15.5 Ultrasound . . . . .	367

15.6 SONAR . . . . .	368
15.6.1 Echolocation . . . . .	368
15.7 Summary . . . . .	369
15.8 Exercises . . . . .	369
<b>16 The Physics of Music - Grade 11</b>	<b>373</b>
16.1 Introduction . . . . .	373
16.2 Standing Waves in String Instruments . . . . .	373
16.3 Standing Waves in Wind Instruments . . . . .	377
16.4 Resonance . . . . .	382
16.5 Music and Sound Quality . . . . .	384
16.6 Summary - The Physics of Music . . . . .	385
16.7 End of Chapter Exercises . . . . .	386
<b>17 Electrostatics - Grade 11</b>	<b>387</b>
17.1 Introduction . . . . .	387
17.2 Forces between charges - Coulomb's Law . . . . .	387
17.3 Electric field around charges . . . . .	392
17.3.1 Electric field lines . . . . .	393
17.3.2 Positive charge acting on a test charge . . . . .	393
17.3.3 Combined charge distributions . . . . .	394
17.3.4 Parallel plates . . . . .	397
17.4 Electrical potential energy and potential . . . . .	400
17.4.1 Electrical potential . . . . .	400
17.4.2 Real-world application: lightning . . . . .	402
17.5 Capacitance and the parallel plate capacitor . . . . .	403
17.5.1 Capacitors and capacitance . . . . .	403
17.5.2 Dielectrics . . . . .	404
17.5.3 Physical properties of the capacitor and capacitance . . . . .	404
17.5.4 Electric field in a capacitor . . . . .	405
17.6 Capacitor as a circuit device . . . . .	406
17.6.1 A capacitor in a circuit . . . . .	406
17.6.2 Real-world applications: capacitors . . . . .	407
17.7 Summary . . . . .	407
17.8 Exercises - Electrostatics . . . . .	407
<b>18 Electromagnetism - Grade 11</b>	<b>413</b>
18.1 Introduction . . . . .	413
18.2 Magnetic field associated with a current . . . . .	413
18.2.1 Real-world applications . . . . .	418
18.3 Current induced by a changing magnetic field . . . . .	420
18.3.1 Real-life applications . . . . .	422
18.4 Transformers . . . . .	423

18.4.1 Real-world applications . . . . .	425
18.5 Motion of a charged particle in a magnetic field . . . . .	425
18.5.1 Real-world applications . . . . .	426
18.6 Summary . . . . .	427
18.7 End of chapter exercises . . . . .	427
<b>19 Electric Circuits - Grade 11</b>	<b>429</b>
19.1 Introduction . . . . .	429
19.2 Ohm's Law . . . . .	429
19.2.1 Definition of Ohm's Law . . . . .	429
19.2.2 Ohmic and non-ohmic conductors . . . . .	431
19.2.3 Using Ohm's Law . . . . .	432
19.3 Resistance . . . . .	433
19.3.1 Equivalent resistance . . . . .	433
19.3.2 Use of Ohm's Law in series and parallel Circuits . . . . .	438
19.3.3 Batteries and internal resistance . . . . .	440
19.4 Series and parallel networks of resistors . . . . .	442
19.5 Wheatstone bridge . . . . .	445
19.6 Summary . . . . .	447
19.7 End of chapter exercise . . . . .	447
<b>20 Electronic Properties of Matter - Grade 11</b>	<b>451</b>
20.1 Introduction . . . . .	451
20.2 Conduction . . . . .	451
20.2.1 Metals . . . . .	453
20.2.2 Insulator . . . . .	453
20.2.3 Semi-conductors . . . . .	454
20.3 Intrinsic Properties and Doping . . . . .	454
20.3.1 Surplus . . . . .	455
20.3.2 Deficiency . . . . .	455
20.4 The p-n junction . . . . .	457
20.4.1 Differences between p- and n-type semi-conductors . . . . .	457
20.4.2 The p-n Junction . . . . .	457
20.4.3 Unbiased . . . . .	457
20.4.4 Forward biased . . . . .	457
20.4.5 Reverse biased . . . . .	458
20.4.6 Real-World Applications of Semiconductors . . . . .	458
20.5 End of Chapter Exercises . . . . .	459
<b>IV Grade 12 - Physics</b>	<b>461</b>
<b>21 Motion in Two Dimensions - Grade 12</b>	<b>463</b>
21.1 Introduction . . . . .	463

21.2 Vertical Projectile Motion . . . . .	463
21.2.1 Motion in a Gravitational Field . . . . .	463
21.2.2 Equations of Motion . . . . .	464
21.2.3 Graphs of Vertical Projectile Motion . . . . .	467
21.3 Conservation of Momentum in Two Dimensions . . . . .	475
21.4 Types of Collisions . . . . .	480
21.4.1 Elastic Collisions . . . . .	480
21.4.2 Inelastic Collisions . . . . .	485
21.5 Frames of Reference . . . . .	490
21.5.1 Introduction . . . . .	490
21.5.2 What is a <i>frame of reference</i> ? . . . . .	491
21.5.3 Why are frames of reference important? . . . . .	491
21.5.4 Relative Velocity . . . . .	491
21.6 Summary . . . . .	494
21.7 End of chapter exercises . . . . .	495
 <b>22 Mechanical Properties of Matter - Grade 12</b>	 <b>503</b>
22.1 Introduction . . . . .	503
22.2 Deformation of materials . . . . .	503
22.2.1 Hooke's Law . . . . .	503
22.2.2 Deviation from Hooke's Law . . . . .	506
22.3 Elasticity, plasticity, fracture, creep . . . . .	508
22.3.1 Elasticity and plasticity . . . . .	508
22.3.2 Fracture, creep and fatigue . . . . .	508
22.4 Failure and strength of materials . . . . .	509
22.4.1 The properties of matter . . . . .	509
22.4.2 Structure and failure of materials . . . . .	509
22.4.3 Controlling the properties of materials . . . . .	509
22.4.4 Steps of Roman Swordsmithing . . . . .	510
22.5 Summary . . . . .	511
22.6 End of chapter exercise . . . . .	511
 <b>23 Work, Energy and Power - Grade 12</b>	 <b>513</b>
23.1 Introduction . . . . .	513
23.2 Work . . . . .	513
23.3 Energy . . . . .	519
23.3.1 External and Internal Forces . . . . .	519
23.3.2 Capacity to do Work . . . . .	520
23.4 Power . . . . .	525
23.5 Important Equations and Quantities . . . . .	529
23.6 End of Chapter Exercises . . . . .	529

<b>24 Doppler Effect - Grade 12</b>	<b>533</b>
24.1 Introduction . . . . .	533
24.2 The Doppler Effect with Sound and Ultrasound . . . . .	533
24.2.1 Ultrasound and the Doppler Effect . . . . .	537
24.3 The Doppler Effect with Light . . . . .	537
24.3.1 The Expanding Universe . . . . .	538
24.4 Summary . . . . .	539
24.5 End of Chapter Exercises . . . . .	539
 <b>25 Colour - Grade 12</b>	<b>541</b>
25.1 Introduction . . . . .	541
25.2 Colour and Light . . . . .	541
25.2.1 Dispersion of white light . . . . .	544
25.3 Addition and Subtraction of Light . . . . .	544
25.3.1 Additive Primary Colours . . . . .	544
25.3.2 Subtractive Primary Colours . . . . .	545
25.3.3 Complementary Colours . . . . .	546
25.3.4 Perception of Colour . . . . .	546
25.3.5 Colours on a Television Screen . . . . .	547
25.4 Pigments and Paints . . . . .	548
25.4.1 Colour of opaque objects . . . . .	548
25.4.2 Colour of transparent objects . . . . .	548
25.4.3 Pigment primary colours . . . . .	549
25.5 End of Chapter Exercises . . . . .	550
 <b>26 2D and 3D Wavefronts - Grade 12</b>	<b>553</b>
26.1 Introduction . . . . .	553
26.2 Wavefronts . . . . .	553
26.3 The Huygens Principle . . . . .	554
26.4 Interference . . . . .	556
26.5 Diffraction . . . . .	557
26.5.1 Diffraction through a Slit . . . . .	558
26.6 Shock Waves and Sonic Booms . . . . .	562
26.6.1 Subsonic Flight . . . . .	563
26.6.2 Supersonic Flight . . . . .	563
26.6.3 Mach Cone . . . . .	566
26.7 End of Chapter Exercises . . . . .	568
 <b>27 Wave Nature of Matter - Grade 12</b>	<b>571</b>
27.1 Introduction . . . . .	571
27.2 de Broglie Wavelength . . . . .	571
27.3 The Electron Microscope . . . . .	574
27.3.1 Disadvantages of an Electron Microscope . . . . .	577

27.3.2 Uses of Electron Microscopes . . . . .	577
27.4 End of Chapter Exercises . . . . .	578
<b>28 Electrodynamics - Grade 12</b>	<b>579</b>
28.1 Introduction . . . . .	579
28.2 Electrical machines - generators and motors . . . . .	579
28.2.1 Electrical generators . . . . .	580
28.2.2 Electric motors . . . . .	582
28.2.3 Real-life applications . . . . .	582
28.2.4 Exercise - generators and motors . . . . .	584
28.3 Alternating Current . . . . .	585
28.3.1 Exercise - alternating current . . . . .	586
28.4 Capacitance and inductance . . . . .	586
28.4.1 Capacitance . . . . .	586
28.4.2 Inductance . . . . .	586
28.4.3 Exercise - capacitance and inductance . . . . .	588
28.5 Summary . . . . .	588
28.6 End of chapter exercise . . . . .	589
<b>29 Electronics - Grade 12</b>	<b>591</b>
29.1 Introduction . . . . .	591
29.2 Capacitive and Inductive Circuits . . . . .	591
29.3 Filters and Signal Tuning . . . . .	596
29.3.1 Capacitors and Inductors as Filters . . . . .	596
29.3.2 LRC Circuits, Resonance and Signal Tuning . . . . .	596
29.4 Active Circuit Elements . . . . .	599
29.4.1 The Diode . . . . .	599
29.4.2 The Light Emitting Diode (LED) . . . . .	601
29.4.3 Transistor . . . . .	603
29.4.4 The Operational Amplifier . . . . .	607
29.5 The Principles of Digital Electronics . . . . .	609
29.5.1 Logic Gates . . . . .	610
29.6 Using and Storing Binary Numbers . . . . .	616
29.6.1 Binary numbers . . . . .	616
29.6.2 Counting circuits . . . . .	617
29.6.3 Storing binary numbers . . . . .	619
<b>30 EM Radiation</b>	<b>625</b>
30.1 Introduction . . . . .	625
30.2 Particle/wave nature of electromagnetic radiation . . . . .	625
30.3 The wave nature of electromagnetic radiation . . . . .	626
30.4 Electromagnetic spectrum . . . . .	626
30.5 The particle nature of electromagnetic radiation . . . . .	629

30.5.1 Exercise - particle nature of EM waves . . . . .	630
30.6 Penetrating ability of electromagnetic radiation . . . . .	631
30.6.1 Ultraviolet(UV) radiation and the skin . . . . .	631
30.6.2 Ultraviolet radiation and the eyes . . . . .	632
30.6.3 X-rays . . . . .	632
30.6.4 Gamma-rays . . . . .	632
30.6.5 Exercise - Penetrating ability of EM radiation . . . . .	633
30.7 Summary . . . . .	633
30.8 End of chapter exercise . . . . .	633
<b>31 Optical Phenomena and Properties of Matter - Grade 12</b>	<b>635</b>
31.1 Introduction . . . . .	635
31.2 The transmission and scattering of light . . . . .	635
31.2.1 Energy levels of an electron . . . . .	635
31.2.2 Interaction of light with metals . . . . .	636
31.2.3 Why is the sky blue? . . . . .	637
31.3 The photoelectric effect . . . . .	638
31.3.1 Applications of the photoelectric effect . . . . .	640
31.3.2 Real-life applications . . . . .	642
31.4 Emission and absorption spectra . . . . .	643
31.4.1 Emission Spectra . . . . .	643
31.4.2 Absorption spectra . . . . .	644
31.4.3 Colours and energies of electromagnetic radiation . . . . .	646
31.4.4 Applications of emission and absorption spectra . . . . .	648
31.5 Lasers . . . . .	650
31.5.1 How a laser works . . . . .	652
31.5.2 A simple laser . . . . .	654
31.5.3 Laser applications and safety . . . . .	655
31.6 Summary . . . . .	656
31.7 End of chapter exercise . . . . .	657
<b>V Exercises</b>	<b>659</b>
<b>32 Exercises</b>	<b>661</b>
<b>VI Essays</b>	<b>663</b>
<b>Essay 1: Energy and electricity. Why the fuss?</b>	<b>665</b>
<b>33 Essay: How a cell phone works</b>	<b>671</b>
<b>34 Essay: How a Physiotherapist uses the Concept of Levers</b>	<b>673</b>
<b>35 Essay: How a Pilot Uses Vectors</b>	<b>675</b>



# Chapter 7

## Geometrical Optics - Grade 10

### 7.1 Introduction

You are indoors on a sunny day. A beam of sunlight through a window lights up a section of the floor. How would you draw this sunbeam? You might draw a series of parallel lines showing the path of the sunlight from the window to the floor. This is not exactly accurate – no matter how hard you look, you will not find unique lines of light in the sunbeam! However, this is a good way to draw light. It is also a good way to model light geometrically, as we will see in this chapter.

We will call these narrow, imaginary lines of light **light rays**. Since light is an electromagnetic wave, you could think of a light ray as the path of a point on the crest of a wave. Or, you could think of a light ray as the path taken by a minuscule particle that carries light. We will always draw them the same way: as straight lines between objects, images, and optical devices.

We can use light rays to model mirrors, lenses, telescopes, microscopes, and prisms. The study of how light interacts with materials is **optics**. When dealing with light rays, we are usually interested in the shape of a material and the angles at which light rays hit it. From these angles, we can work out, for example, the distance between an object and its reflection. We therefore refer to this kind of optics as **geometrical optics**.

### 7.2 Light Rays

In physics we use the idea of a *light ray* to indicate the direction that light travels. Light rays are lines with arrows and are used to show the path that light travels. In Figure 7.1, the light rays from the object enter the eye and the eye sees the object.

The most important thing to remember is that we can only see an object when light from the object enters our eyes. The object must be a source of light (for example a light bulb) or else it must reflect light from a source (for example the moon), and the reflected light enters our eyes.

**Important:** We cannot see an object unless light from that object enters our eyes.



**Definition: Light ray**

Light rays are straight lines with arrows to show the path of light.



**Important:** Light rays are not real. They are merely used to show the path that light travels.

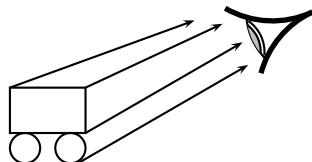


Figure 7.1: We can only see an object when light from that object enters our eyes. We draw light as lines with arrows to show the direction the light travels. When the light travels from the object to the eye, the eye can see the object.

### Activity :: Investigation : Light travels in straight lines

#### Apparatus:

You will need a candle, matches and three sheets of paper.

#### Method:

1. Make a small hole in the middle of each of the three sheets of paper.
2. Light the candle.
3. Look at the burning candle through the hole in the first sheet of paper.
4. Place the second sheet of paper between you and the candle so that you can still see the candle through the holes.
5. Now do the same with the third sheet so that you can still see the candle. The sheets of paper must not touch each other.

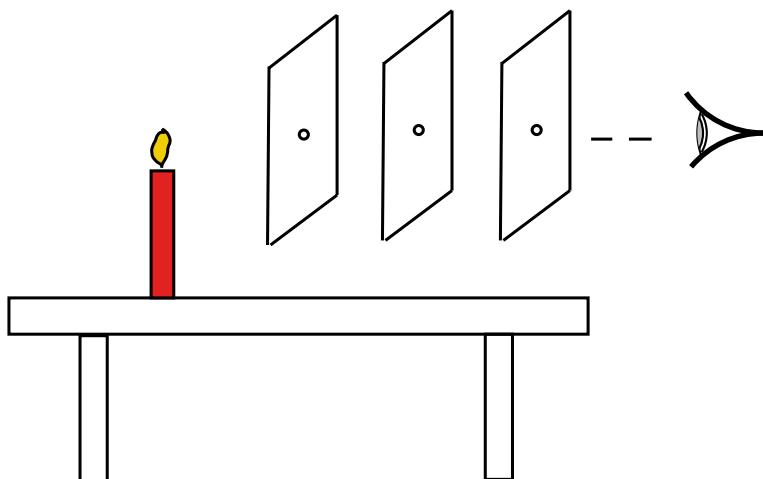


Figure 7.2: Light travels in straight lines

6. What do you notice about the holes in the paper?

#### Conclusions:

In the investigation you will notice that the holes in the paper need to be in a straight line. This shows that light travels in a straight line. We cannot see around corners. This also proves that light does not bend around a corner, but travels straight.

### Activity :: Investigation : Light travels in straight lines

On a sunny day, stand outside and look at something in the distance, for example a tree, a flower or a car. From what we have learnt, we can see the tree, flower or car because light from the object is entering our eye. Now take a sheet of paper and hold it about 20 cm in front of your face. Can you still see the tree, flower or car? Why not?

Figure 7.3 shows that a sheet of paper in front of your eye prevents light rays from reaching your eye.

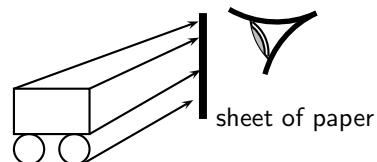
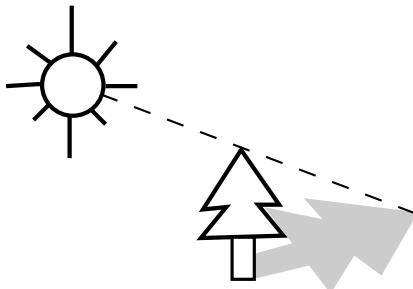


Figure 7.3: The sheet of paper prevents the light rays from reaching the eye, and the eye cannot see the object.

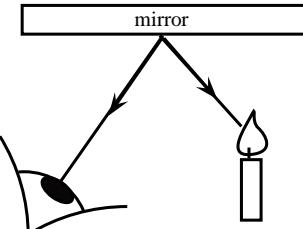
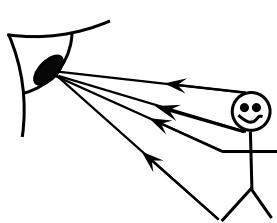
### 7.2.1 Shadows

Objects cast shadows when light shines on them. This is more evidence that light travels in straight lines. The picture below shows how shadows are formed.



### 7.2.2 Ray Diagrams

A ray diagram is a drawing that shows the path of light rays. Light rays are drawn using straight lines and arrow heads. The figure below shows some examples of ray diagrams.



#### Exercise: Light Rays

1. Are light rays real? Explain.
2. Give evidence to support the statement: "Light travels in straight lines". Draw a ray diagram to prove this.
3. You are looking at a burning candle. Draw the path of light that enables you to see that candle.

## 7.3 Reflection

When you smile into a mirror, you see your own face smiling back at you. This is caused by the reflection of light rays on the mirror. Reflection occurs when a light ray bounces off a surface.

### 7.3.1 Terminology

In Chapters 5 and 6 we saw that when a pulse or wave strikes a surface it is *reflected*. This means that waves bounce off things. Sound waves bounce off walls, light waves bounce off mirrors, radar waves bounce off aeroplanes and it can explain how bats can fly at night and avoid things as thin as telephone wires. The phenomenon of reflection is a very important and useful one.

We will use the following terminology. The incoming light ray is called the **incident ray**. The light ray moving away from the surface is the **reflected ray**. The most important characteristic of these rays is their angles in relation to the reflecting surface. These angles are measured with respect to the normal of the surface. The **normal** is an imaginary line perpendicular to the surface. The **angle of incidence**,  $\theta_i$  is measured between the incident ray and the surface normal. The **angle of reflection**,  $\theta_r$  is measured between the reflected ray and the surface normal. This is shown in Figure 7.4.

When a ray of light is reflected, the reflected ray lies in the same plane as the incident ray and the normal. This plane is called the **plane of incidence** and is shown in Figure 7.5.

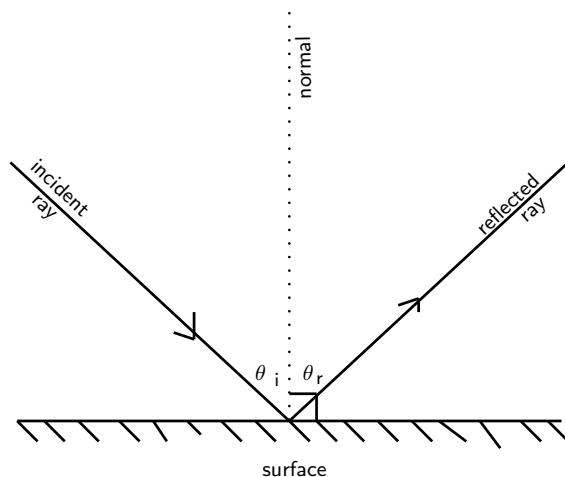


Figure 7.4: The angles of incidence and reflection are measured with respect to the surface normal.

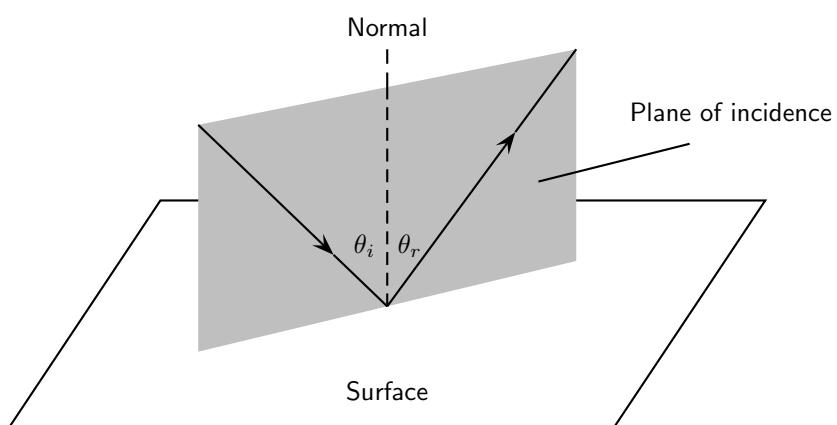


Figure 7.5: The plane of incidence is the plane including the incident ray, reflected ray, and the surface normal.

### 7.3.2 Law of Reflection

The **Law of Reflection** states that the angles of incidence and reflection are always equal and that the reflected ray always lies in the plane of incidence.


**Definition: Law of Reflection**

The Law of Reflection states that the angle of incidence is equal to the angle of reflection.

$$\theta_i = \theta_r$$

The simplest example of the law of incidence is if the angle of incidence is  $0^\circ$ . In this case, the angle of reflection is also  $0^\circ$ . You see this when you look straight into a mirror.

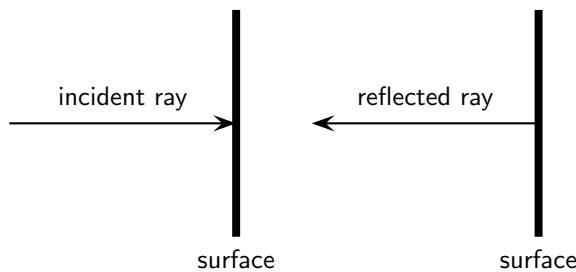


Figure 7.6: When a wave strikes a surface at right angles to the surface, then the wave is reflected directly back.

If the angle of incidence is not  $0^\circ$ , then the angle of reflection is also not  $0^\circ$ . For example, if a light strikes a surface at  $60^\circ$  to the surface normal, then the angle that the reflected ray makes with the surface normal is also  $60^\circ$  as shown in Figure 7.7.

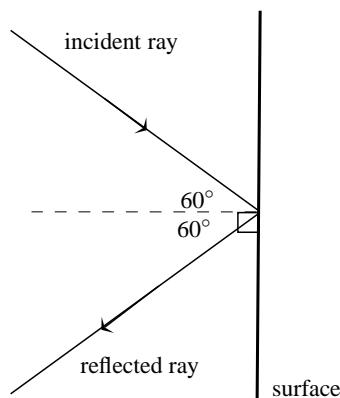


Figure 7.7: Ray diagram showing angle of incidence and angle of reflection. The Law of Reflection states that when a light ray reflects off a surface, the angle of reflection  $\theta_r$  is the same as the angle of incidence  $\theta_i$ .


**Worked Example 31: Law of Reflection**

**Question:** An incident ray strikes a smooth reflective surface at an angle of  $33^\circ$  to the surface normal. Calculate the angle of reflection.

**Answer**

**Step 1 : Determine what is given and what is required**

We are given the angle between the incident ray and the surface normal. This is the angle of incidence.

We are required to calculate the angle of reflection.

**Step 2 : Determine how to approach the problem**

We can use the Law of Reflection, which states that the angle of incidence is equal to the angle of reflection.

**Step 3 : Calculate the angle of reflection**

We are given the angle of incidence to be  $33^\circ$ . Therefore, the angle of reflection is also  $33^\circ$ .

### 7.3.3 Types of Reflection

The Law of Reflection is true for any surface. Does this mean that when parallel rays approach a surface, the reflected rays will also be parallel? This depends on the texture of the reflecting surface.

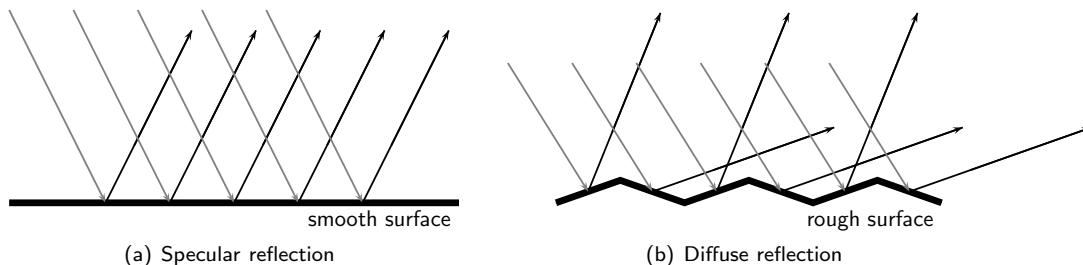


Figure 7.8: Specular and diffuse reflection.

#### Specular Reflection

Figure 7.8(a) shows a surface that is flat and even. Parallel incident light rays hit the smooth surface and parallel reflected light rays leave the surface. This type of reflection is called **specular reflection**. Specular reflection occurs when rays are reflected from a smooth, shiny surface. The normal to the surface is the same at every point on the surface. Parallel incident rays become parallel reflected rays. When you look in a mirror, the image you see is formed by specular reflection.

#### Diffuse Reflection

Figure 7.8(b) shows a surface with bumps and curves. When multiple rays hit this uneven surface, **diffuse reflection** occurs. The incident rays are parallel but the reflected rays are not. Each point on the surface has a different normal. This means the angle of incidence is different at each point. Then according to the Law of Reflection, each angle of reflection is different. Diffuse reflection occurs when light rays are reflected from bumpy surfaces. You can still see a reflection as long as the surface is not too bumpy. Diffuse reflection enables us to see all objects that are not sources of light.

#### Activity :: Experiment : Specular and Diffuse Reflection

A bouncing ball can be used to demonstrate the basic difference between specular and diffuse reflection.

**Aim:**

To demonstrate and compare specular and diffuse reflection.

**Apparatus:**

You will need:

1. a small ball (a tennis ball or a table tennis ball is perfect)
2. a smooth surface, like the floor inside the classroom
3. a very rough surface, like a rocky piece of ground

**Method:**

1. Bounce the ball on the smooth floor and observe what happens.
2. Bounce the ball on the rough ground floor and observe what happens.
3. What do you observe?

4. What is the difference between the two surfaces?

**Conclusions:**

You should have seen that the ball bounces (is reflected off the floor) in a predictable manner off the smooth floor, but bounces unpredictably on the rough ground.

The ball can be seen to be a ray of light and the floor or ground is the reflecting surface. For specular reflection (smooth surface), the ball bounces predictably. For diffuse reflection (rough surface), the ball bounces unpredictably.

---

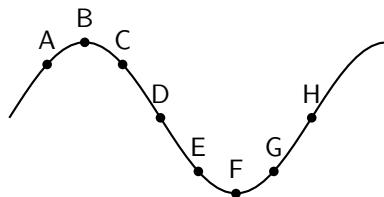


---



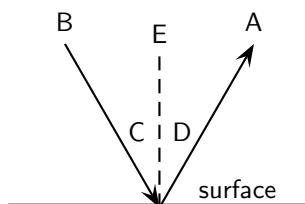
**Exercise: Reflection**

1. The diagram shows a curved surface. Draw normals to the surface at the marked points.

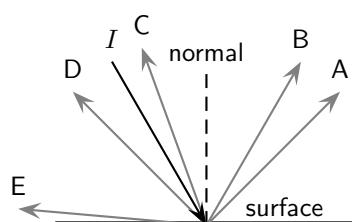


2. In the diagram, label the following:

- (a) normal
- (b) angle of incidence
- (c) angle of reflection
- (d) incident ray
- (e) reflected ray



3. State the Law of Reflection. Draw a diagram, label the appropriate angles and write a mathematical expression for the Law of Reflection.
4. Draw a ray diagram to show the relationship between the angle of incidence and the angle of reflection.
5. The diagram shows an incident ray *I*. Which of the other 5 rays (A, B, C, D, E) best represents the reflected ray of *I*?



6. A ray of light strikes a surface at  $15^\circ$  to the surface normal. Draw a ray diagram showing the incident ray, reflected ray and surface normal. Calculate the angles of incidence and reflection and fill them in your diagram.

7. A ray of light leaves a surface at  $45^\circ$  to the surface normal. Draw a ray diagram showing the incident ray, reflected ray and surface normal. Calculate the angles of incidence and reflection and fill them in on your diagram.
  8. A ray of light strikes a surface at  $25^\circ$  to the surface. Draw a ray diagram showing the incident ray, reflected ray and surface normal. Calculate the angles of incidence and reflection and fill them in on your diagram.
  9. A ray of light leaves a surface at  $65^\circ$  to the surface. Draw a ray diagram showing the incident ray, reflected ray and surface normal. Calculate the angles of incidence and reflection and fill them in on your diagram.
  10. If the incident ray, the reflected ray and the surface normal do not fall on the same plane, will the angle of incidence equal the angle of reflection?
  11. Explain the difference between specular and diffuse reflection.
  12. We see an object when the light that is reflected by the object enters our eyes. Do you think the reflection by most objects is specular reflection or diffuse reflection? Explain.
  13. A beam of light (for example from a torch) is generally not visible at night, as it travels through air. Try this for yourself. However, if you shine the torch through dust, the beam is visible. Explain why this happens.
  14. If a torch beam is shone across a classroom, only students in the direct line of the beam would be able to see that the torch is shining. However, if the beam strikes a wall, the entire class will be able to see the spot made by the beam on the wall. Explain why this happens.
  15. A scientist looking into a flat mirror hung perpendicular to the floor cannot see her feet but she can see the hem of her lab coat. Draw a ray diagram to help explain the answers to the following questions:
    - (a) Will she be able to see her feet if she backs away from the mirror?
    - (b) What if she moves towards the mirror?
- 

## 7.4 Refraction

In the previous sections we studied light reflecting off various surfaces. What happens when light passes *through* a medium? Like all waves, the speed of light is dependent on the medium in which it is travelling. When light moves from one medium into another (for example, from air to glass), the speed of light changes. The effect is that the light ray passing into a new medium is **refracted**, or bent. Refraction is therefore the bending of light as it moves from one optical medium to another.



### Definition: Refraction

Refraction is the bending of light that occurs because light travels at different speeds in different materials.

When light travels from one medium to another, it will be bent away from its original path. When it travels from an optically dense medium like water or glass to a less dense medium like air, it will be refracted away from the normal (Figure 7.9). Whereas, if it travels from a less dense medium to a denser one, it will be refracted towards the normal (Figure 7.10).

Just as we defined an angle of reflection in the previous section, we can similarly define an angle of refraction as the angle between the surface normal and the refracted ray. This is shown in Figure 7.11.

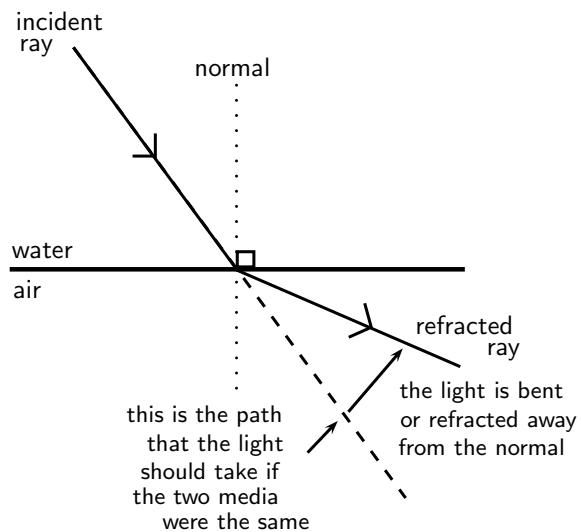


Figure 7.9: Light is moving from an optically dense medium to an optically less dense medium. Light is refracted away from the normal.

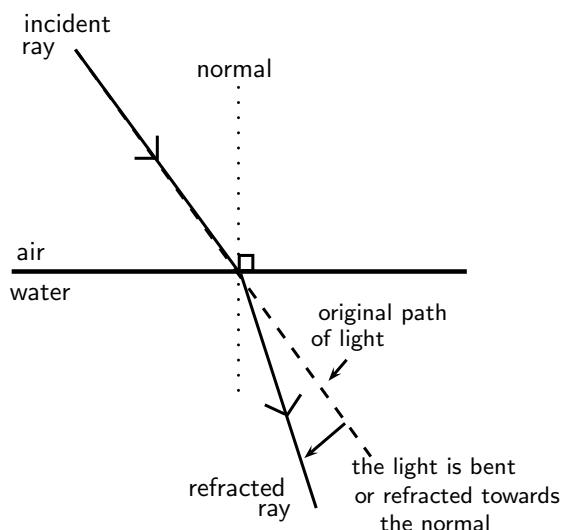


Figure 7.10: Light is moving from an optically less dense medium to an optically denser medium. Light is refracted towards the normal.

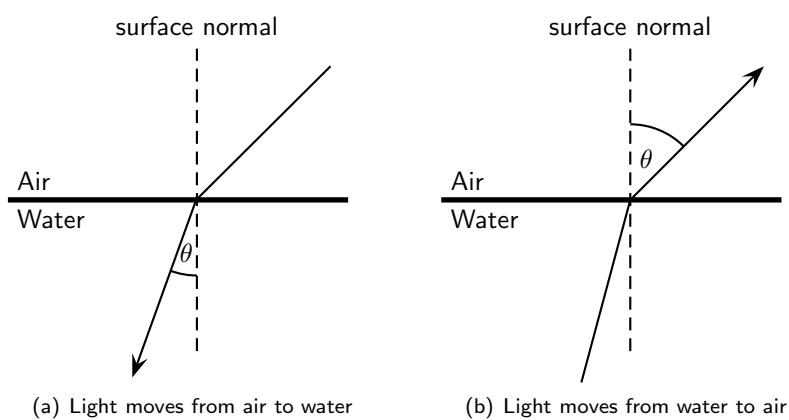


Figure 7.11: Light moving from one medium to another bends towards or away from the surface normal. The angle of refraction  $\theta$  is shown.

### 7.4.1 Refractive Index

Which is easier to travel through, air or water? People usually travel faster through air. So does light! The speed of light and therefore the degree of bending of the light depends on the *refractive index* of material through which the light passes. The refractive index (symbol  $n$ ) is the ratio of the speed of light in a vacuum to its speed in the material. You can think of the refractive index as a measure of how difficult it is for light to get through a material.



#### Definition: Refractive Index

The refractive index of a material is the ratio of the speed of light in a vacuum to its speed in the medium.



The symbol  $c$  is used to represent the speed of light in a vacuum.

$$c = 299\ 792\ 485 \text{ m} \cdot \text{s}^{-1}$$

For purposes of calculation, we use  $3 \times 10^8 \text{ m} \cdot \text{s}^{-1}$ . A vacuum is a region with no matter in it, not even air. However, the speed of light in air is very close to that in a vacuum.



#### Definition: Refractive Index

The refractive index (symbol  $n$ ) of a material is the ratio of the speed of light in a vacuum to its speed in the material and gives an indication of how difficult it is for light to get through the material.

$$n = \frac{c}{v}$$

where

$n$  = refractive index (no unit)

$c$  = speed of light in a vacuum ( $3,00 \times 10^8 \text{ m} \cdot \text{s}^{-1}$ )

$v$  = speed of light in a given medium ( $\text{m} \cdot \text{s}^{-1}$ )



#### Extension: Refractive Index and Speed of Light

Using

$$n = \frac{c}{v}$$

we can also examine how the speed of light changes in different media, because the speed of light in a vacuum ( $c$ ) is constant.

If the refractive index  $n$  increases, the speed of light in the material  $v$  must decrease. Light therefore travels slowly through materials of high  $n$ .

Table 7.4.1 shows refractive indices for various materials. Light travels slower in any material than it does in a vacuum, so all values for  $n$  are greater than 1.

### 7.4.2 Snell's Law

Now that we know that the degree of bending, or the angle of refraction, is dependent on the refractive index of a medium, how do we calculate the angle of refraction?

The angles of incidence and refraction when light travels from one medium to another can be calculated using Snell's Law.

Medium	Refractive Index
Vacuum	1
Helium	1,000036
Air*	1,0002926
Carbon dioxide	1,00045
Water: Ice	1,31
Water: Liquid (20°C)	1,333
Acetone	1,36
Ethyl Alcohol (Ethanol)	1,36
Sugar solution (30%)	1,38
Fused quartz	1,46
Glycerine	1,4729
Sugar solution (80%)	1,49
Rock salt	1,516
Crown Glass	1,52
Sodium chloride	1,54
Polystyrene	1,55 to 1,59
Bromine	1,661
Sapphire	1,77
Glass (typical)	1,5 to 1,9
Cubic zirconia	2,15 to 2,18
Diamond	2,419
Silicon	4,01

Table 7.1: Refractive indices of some materials.  $n_{\text{air}}$  is calculated at STP and all values are determined for yellow sodium light which has a wavelength of 589,3 nm.



### Definition: Snell's Law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

where

$n_1$  = Refractive index of material 1

$n_2$  = Refractive index of material 2

$\theta_1$  = Angle of incidence

$\theta_2$  = Angle of refraction

Remember that angles of incidence and refraction are measured from the normal, which is an imaginary line perpendicular to the surface.

Suppose we have two media with refractive indices  $n_1$  and  $n_2$ . A light ray is incident on the surface between these materials with an **angle of incidence**  $\theta_1$ . The refracted ray that passes through the second medium will have an **angle of refraction**  $\theta_2$ .



### Worked Example 32: Using Snell's Law

**Question:** A light ray with an angle of incidence of  $35^\circ$  passes from water to air. Find the angle of refraction using Snell's Law and Table 7.4.1. Discuss the meaning of your answer.

#### Answer

##### Step 1 : Determine the refractive indices of water and air

From Table 7.4.1, the refractive index is 1,333 for water and about 1 for air. We know the angle of incidence, so we are ready to use Snell's Law.

##### Step 2 : Substitute values

According to Snell's Law:

$$\begin{aligned} n_1 \sin \theta_1 &= n_2 \sin \theta_2 \\ 1,33 \sin 35^\circ &= 1 \sin \theta_2 \\ \sin \theta_2 &= 0,763 \\ \theta_2 &= 49,7^\circ. \end{aligned}$$

### Step 3 : Discuss the answer

The light ray passes from a medium of high refractive index to one of low refractive index. Therefore, the light ray is bent away from the normal.



### Worked Example 33: Using Snell's Law

**Question:** A light ray passes from water to diamond with an angle of incidence of  $75^\circ$ . Calculate the angle of refraction. Discuss the meaning of your answer.

#### Answer

#### Step 1 : Determine the refractive indices of water and air

From Table 7.4.1, the refractive index is 1,333 for water and 2,42 for diamond. We know the angle of incidence, so we are ready to use Snell's Law.

#### Step 2 : Substitute values and solve

According to Snell's Law:

$$\begin{aligned} n_1 \sin \theta_1 &= n_2 \sin \theta_2 \\ 1,33 \sin 75^\circ &= 2,42 \sin \theta_2 \\ \sin \theta_2 &= 0,531 \\ \theta_2 &= 32,1^\circ. \end{aligned}$$

### Step 3 : Discuss the answer

The light ray passes from a medium of low refractive index to one of high refractive index. Therefore, the light ray is bent towards the normal.

If

$$n_2 > n_1$$

then from Snell's Law,

$$\sin \theta_1 > \sin \theta_2.$$

For angles smaller than  $90^\circ$ ,  $\sin \theta$  increases as  $\theta$  increases. Therefore,

$$\theta_1 > \theta_2.$$

This means that the angle of incidence is greater than the angle of refraction and the light ray is bent toward the normal.

Similarly, if

$$n_2 < n_1$$

then from Snell's Law,

$$\sin \theta_1 < \sin \theta_2.$$

For angles smaller than  $90^\circ$ ,  $\sin \theta$  increases as  $\theta$  increases. Therefore,

$$\theta_1 < \theta_2.$$

This means that the angle of incidence is less than the angle of refraction and the light ray is away toward the normal.

Both these situations can be seen in Figure 7.12.

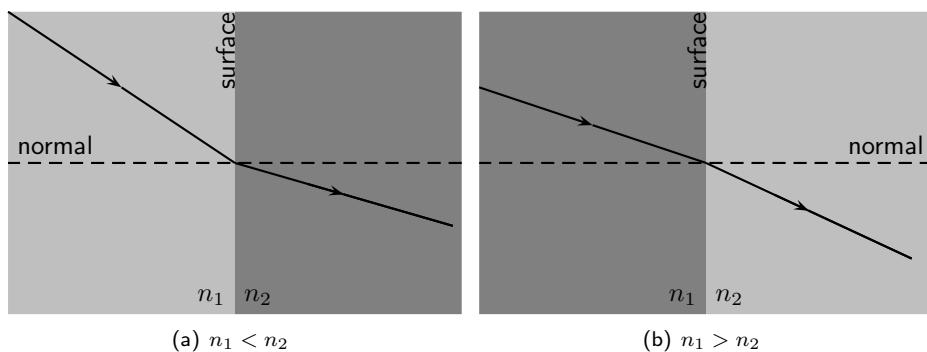


Figure 7.12: Refraction of two light rays. (a) A ray travels from a medium of low refractive index to one of high refractive index. The ray is bent towards the normal. (b) A ray travels from a medium with a high refractive index to one with a low refractive index. The ray is bent away from the normal.

What happens to a ray that lies along the normal line? In this case, the angle of incidence is  $0^\circ$  and

$$\begin{aligned}\sin \theta_2 &= \frac{n_1}{n_2} \sin \theta_1 \\ &= 0 \\ \therefore \theta_2 &= 0.\end{aligned}$$

This shows that if the light ray is incident at  $0^\circ$ , then the angle of refraction is also  $0^\circ$ . The ray passes through the surface unchanged, i.e. no refraction occurs.

### Activity :: Investigation : Snell's Law 1

The angles of incidence and refraction were measured in five unknown media and recorded in the table below. Use your knowledge about Snell's Law to identify each of the unknown media A - E. Use Table 7.4.1 to help you.

Medium 1	$n_1$	$\theta_1$	$\theta_2$	$n_2$	Unknown Medium
Air	1,000036	38	11,6	?	A
Air	1,000036	65	38,4	?	B
Vacuum	1	44	0,419	?	C
Air	1,000036	15	29,3	?	D
Vacuum	1	20	36,9	?	E

### Activity :: Investigation : Snell's Law 2

Zingi and Tumi performed an investigation to identify an unknown liquid. They shone a beam of light into the unknown liquid, varying the angle of incidence and recording the angle of refraction. Their results are recorded in the following table:

Angle of Incidence	Angle of Refraction
0,0°	0,00°
5,0°	3,76°
10,0°	7,50°
15,0°	11,2°
20,0°	14,9°
25,0°	18,5°
30,0°	22,1°
35,0°	25,5°
40,0°	28,9°
45,0°	32,1°
50,0°	35,2°
55,0°	38,0°
60,0°	40,6°
65,0°	43,0°
70,0°	?
75,0°	?
80,0°	?
85,0°	?

1. Write down an aim for the investigation.
  2. Make a list of all the apparatus they used.
  3. Identify the unknown liquid.
  4. Predict what the angle of refraction will be for 70°, 75°, 80° and 85°.
- 

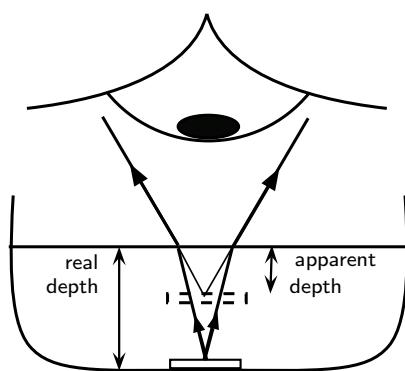
### 7.4.3 Apparent Depth

Imagine a coin on the bottom of a shallow pool of water. If you reach for the coin, you will miss it because the light rays from the coin are refracted at the water's surface.

Consider a light ray that travels from an underwater object to your eye. The ray is refracted at the water surface and then reaches your eye. Your eye does not know Snell's Law; it assumes light rays travel in straight lines. Your eye therefore sees the image of the at coin shallower location. This shallower location is known as the *apparent depth*.

The refractive index of a medium can also be expressed as

$$n = \frac{\text{real depth}}{\text{apparent depth}}.$$



**Question:** A coin is placed at the bottom of a 40 cm deep pond. The refractive index for water is 1,33. How deep does the coin appear to be?

**Answer**

**Step 1 : Identify what is given and what is asked**

$$n = 1,33$$

$$\text{real depth} = 40 \text{ cm}$$

$$\text{apparent depth} = ?$$

**Step 2 : Substitute values and find answer**

$$\begin{aligned} n &= \frac{\text{real depth}}{\text{apparent depth}} \\ 1,33 &= \frac{40}{x} \\ x &= \frac{40}{1,33} = 30,08 \text{ cm} \end{aligned}$$

The coin appears to be 30,08 cm deep.



### Worked Example 35: Apparent Depth 2

**Question:** A R1 coin appears to be 7 cm deep in a colourless liquid. The depth of the liquid is 10,43 cm.

1. Determine the refractive index of the liquid.
2. Identify the liquid.

**Answer**

**Step 1 : Identify what is given and what is asked**

$$\text{real depth} = 7 \text{ cm}$$

$$\text{apparent depth} = 10,43 \text{ cm}$$

$$n = ?$$

Identify the liquid.

**Step 2 : Calculate refractive index**

$$\begin{aligned} n &= \frac{\text{real depth}}{\text{apparent depth}} \\ &= \frac{10,43}{7} \\ &= 1,49 \end{aligned}$$

**Step 3 : Identify the liquid**

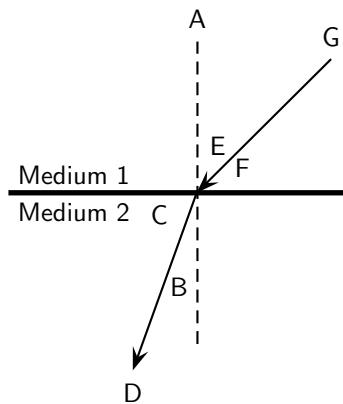
Use Table 7.4.1. The liquid is an 80% sugar solution.



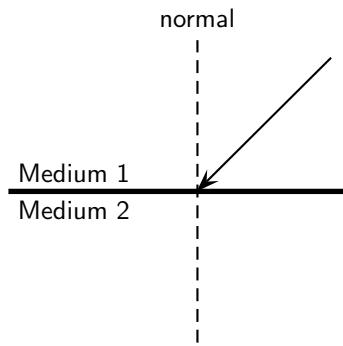
### Exercise: Refraction

1. Explain refraction in terms of a change of wave speed in different media.
2. In the diagram, label the following:
  - (a) angle of incidence
  - (b) angle of refraction

- (c) incident ray
- (d) refracted ray
- (e) normal



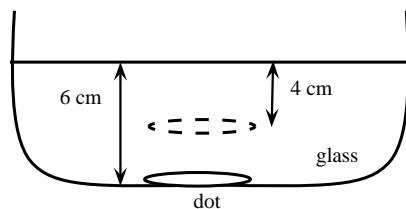
3. What is *the angle of refraction*?
4. Describe what is meant by *the refractive index of a medium*.
5. State Snell's Law.
6. In the diagram, a ray of light strikes the interface between two media.



Draw what the refracted ray would look like if:

- (a) medium 1 had a higher refractive index than medium 2.
  - (b) medium 1 had a lower refractive index than medium 2.
7. Light travels from a region of glass into a region of glycerine, making an angle of incidence of  $40^\circ$ .
    - (a) Describe the path of the light as it moves into the glycerine.
    - (b) Calculate the angle of refraction.
  8. A ray of light travels from silicon to water. If the ray of light in the water makes an angle of  $69^\circ$  to the surface normal, what is the angle of incidence in the silicon?
  9. Light travels from a medium with  $n = 1,25$  into a medium of  $n = 1,34$ , at an angle of  $27^\circ$  from the interface normal.
    - (a) What happens to the speed of the light? Does it increase, decrease, or remain the same?
    - (b) What happens to the wavelength of the light? Does it increase, decrease, or remain the same?
    - (c) Does the light bend towards the normal, away from the normal, or not at all?
  10. Light travels from a medium with  $n = 1,63$  into a medium of  $n = 1,42$ .
    - (a) What happens to the speed of the light? Does it increase, decrease, or remain the same?
    - (b) What happens to the wavelength of the light? Does it increase, decrease, or remain the same?

- (c) Does the light bend towards the normal, away from the normal, or not at all?
11. Light is incident on a glass prism. The prism is surrounded by air. The angle of incidence is  $23^\circ$ . Calculate the angle of reflection and the angle of refraction.
  12. Light is refracted at the interface between air and an unknown medium. If the angle of incidence is  $53^\circ$  and the angle of refraction is  $37^\circ$ , calculate the refractive index of the unknown, second medium.
  13. A coin is placed in a bowl of acetone ( $n = 1.36$ ). The coin appears to be 10 cm deep. What is the depth of the acetone?
  14. A dot is drawn on a piece of paper and a glass prism placed on the dot according to the diagram.



- (a) Use the information supplied to determine the refractive index of glass.
  - (b) Draw a ray diagram to explain how the image of the dot is above where the dot really is.
  15. Light is refracted at the interface between a medium of refractive index 1.5 and a second medium of refractive index 2.1. If the angle of incidence is  $45^\circ$ , calculate the angle of refraction.
  16. A ray of light strikes the interface between air and diamond. If the incident ray makes an angle of  $30^\circ$  with the interface, calculate the angle made by the refracted ray with the interface.
  17. **Challenge Question:** What values of  $n$  are physically impossible to achieve? Explain your answer. The values provide the limits of possible refractive indices.
  18. **Challenge Question:** You have been given a glass beaker full of an unknown liquid. How would you identify what the liquid is? You have the following pieces of equipment available for the experiment: a laser, a protractor, a ruler, a pencil, and a reference guide containing optical properties of various liquids.
- 

## 7.5 Mirrors

A mirror is a highly reflective surface. The most common mirrors are flat and are known as **plane mirrors**. Household mirrors are plane mirrors. They are made of a flat piece of glass with a thin layer of silver nitrate or aluminium on the back. However, other mirrors are curved and are either **convex mirrors** or are **concave mirrors**. The reflecting properties of all three types of mirrors will be discussed in this section.

### 7.5.1 Image Formation



#### Definition: Image

An image is a representation of an object formed by a mirror or lens. Light from the image is seen.

If you place a candle in front of a mirror, you now see two candles. The actual, physical candle is called the **object** and the picture you see in the mirror is called the **image**. The **object** is the source of the incident rays. The **image** is the picture that is formed by the reflected rays.

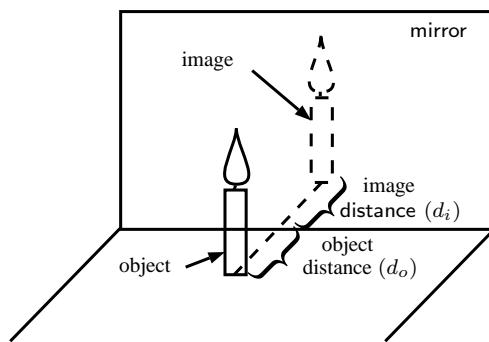


Figure 7.13: An object formed in a mirror is real and upright.

The object could be an actual source that emits light, such as a light bulb or a candle. More commonly, the object reflects light from another source. When you look at your face in the mirror, your face does not emit light. Instead, light from a light bulb or from the sun reflects off your face and then hits the mirror. However, in working with light rays, it is easiest to pretend the light is coming from the object.

An image formed by reflection may be real or virtual. A **real** image occurs when light rays actually intersect at the image. A real image is inverted, or upside down. A **virtual** image occurs when light rays do not actually meet at the image. Instead, you "see" the image because your eye projects light rays backward. You are fooled into seeing an image! A virtual image is erect, or right side up (upright).

You can tell the two types apart by putting a screen at the location of the image. A real image can be formed on the screen because the light rays actually meet there. A virtual image cannot be seen on a screen, since it is not really there.

To describe objects and images, we need to know their locations and their sizes. The distance from the mirror to the object is the **object distance**,  $d_o$ .

The distance from the mirror to the image is the **image distance**,  $d_i$ .

### 7.5.2 Plane Mirrors

---

#### Activity :: Investigation : Image formed by a mirror

1. Stand one step away from a large mirror
  2. What do you observe in the mirror? This is called your image.
  3. What size is your image? Bigger, smaller or the same size as you?
  4. How far is your image from you? How far is your image from the mirror?
  5. Is your image upright or upside down?
  6. Take one step backwards. What does your image do? How far are you away from your image?
  7. Lift your left arm. Which arm does your image lift?
- 

When you look into a mirror, you see an *image* of yourself.

The image created in the mirror has the following properties:

1. The image is *virtual*.

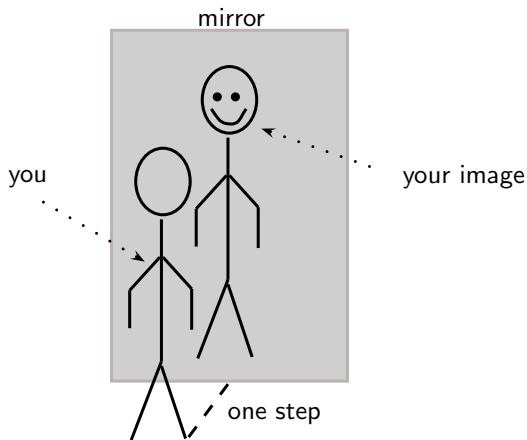


Figure 7.14: An image in a mirror is virtual, upright, the same size and laterally inverted.

2. The image is the same distance behind the mirror as the object is in front of the mirror.
3. The image is *laterally* inverted. This means that the image is inverted from side to side.
4. The image is the same size as the object.
5. The image is upright.

Virtual images are images formed in places where light does not really reach. Light does not really pass through the mirror to create the image; it only appears to an observer as though the light were coming from behind the mirror. Whenever a mirror creates an image which is virtual, the image will always be located behind the mirror where light does not really pass.



#### **Definition: Virtual Image**

A virtual image is upright, on the opposite side of the mirror as the object, and light does not actually reach it.

### 7.5.3 Ray Diagrams

We draw *ray diagrams* to predict the image that is formed by a plane mirror. A ray diagram is a geometrical picture that is used for analyzing the images formed by mirrors and lenses. We draw a few characteristic rays from the object to the mirror. We then follow ray-tracing rules to find the path of the rays and locate the image.



**Important:** A mirror obeys the Law of Reflection.

The ray diagram for the image formed by a plane mirror is the simplest possible ray diagram. Figure 7.15 shows an object placed in front of a plane mirror. It is convenient to have a central line that runs perpendicular to the mirror. This imaginary line is called the **principal axis**.



#### **Important: Ray diagrams**

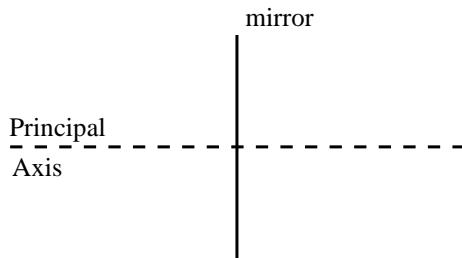
The following should be remembered when drawing ray diagrams:

1. Objects are represented by arrows. The length of the arrow represents the height of the object.
2. If the arrow points upwards, then the object is described as upright or erect. If the arrow points downwards then the object is described as inverted.
3. If the object is real, then the arrow is drawn with a solid line. If the object is virtual, then the arrow is drawn with a dashed line.

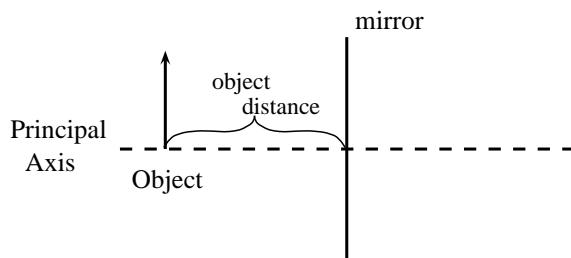
**Method: Ray Diagrams for Plane Mirrors**

Ray diagrams are used to find the position and size and whether the image is real or virtual.

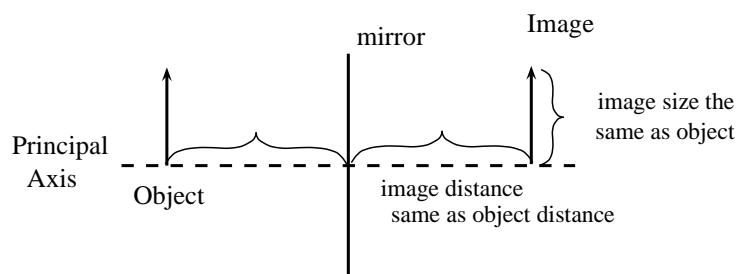
1. Draw the plane mirror as a straight line on a principal axis.



2. Draw the object as an arrow in front of the mirror.

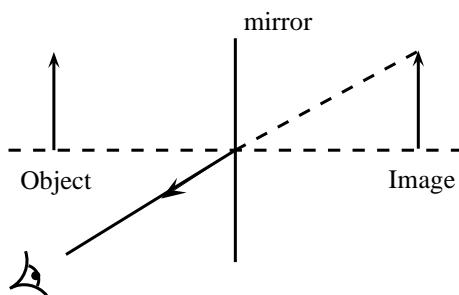


3. Draw the image of the object, by using the principle that the image is placed at the same distance behind the mirror that the object is in front of the mirror. The image size is also the same as the object size.

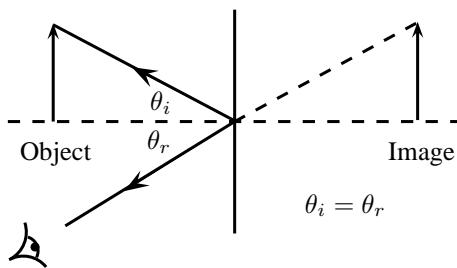


4. Place a dot at the point the eye is located.

5. Pick one point on the image and draw the reflected ray that travels to the eye as it sees this point. Remember to add an arrowhead.



6. Draw the incident ray for light traveling from the corresponding point on the object to the mirror, such that the law of reflection is obeyed.



7. Continue for other extreme points on the object.

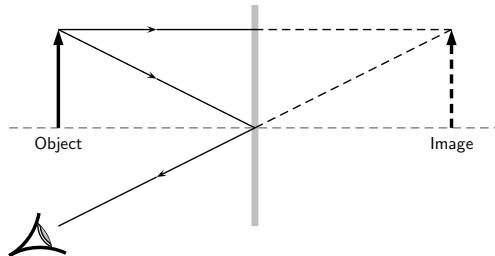


Figure 7.15: Ray diagram to predict the image formed by a plane mirror.

Suppose a light ray leaves the top of the object traveling parallel to the principal axis. The ray will hit the mirror at an angle of incidence of 0 degrees. We say that the ray hits the mirror *normally*. According to the law of reflection, the ray will be reflected at 0 degrees. The ray then bounces back in the same direction. We also project the ray back behind the mirror because this is what your eye does.

Another light ray leaves the top of the object and hits the mirror at its centre. This ray will be reflected at the same angle as its angle of incidence, as shown. If we project the ray backward behind the mirror, it will eventually cross the projection of the first ray we drew. We have found the location of the image! It is a virtual image since it appears in an area that light cannot actually reach (behind the mirror). You can see from the diagram that the image is erect and is the same size as the object. This is exactly as we expected.

We use a dashed line to indicate that the image is virtual.

#### 7.5.4 Spherical Mirrors

The second class of mirrors that we will look at are spherical mirrors. These mirrors are called spherical mirrors because if you take a sphere and cut it as shown in Figure 7.16 and then polish the inside of one and the outside of the other, you will get a *concave mirror* and *convex mirror* as shown. These two mirrors will be studied in detail.

The centre of curvature is the point at the centre of the sphere and describes how big the sphere is.

#### 7.5.5 Concave Mirrors

The first type of curved mirror we will study are concave mirrors. Concave mirrors have the shape shown in Figure 7.17. As with a plane mirror, the principal axis is a line that is perpendicular to the centre of the mirror.

If you think of light reflecting off a concave mirror, you will immediately see that things will look very different compared to a plane mirror. The easiest way to understand what will happen is to draw a ray diagram and work out where the images will form. Once we have done that it is easy to see what properties the image has.

First we need to define a very important characteristic of the mirror. We have seen that the centre of curvature is the centre of the sphere from which the mirror is cut. We then define that

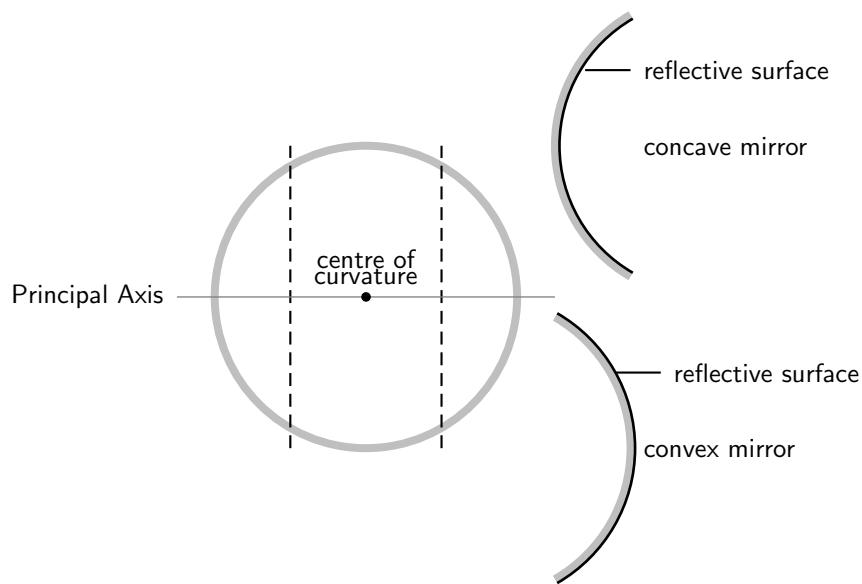


Figure 7.16: When a sphere is cut and then polished to a reflective surface on the inside a concave mirror is obtained. When the outside is polished to a reflective surface, a convex mirror is obtained.

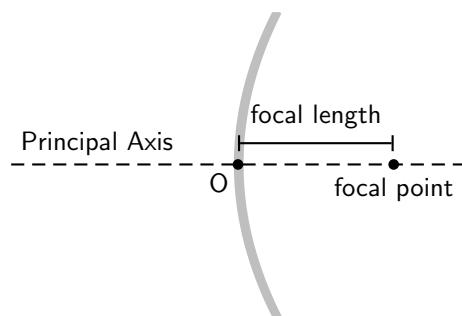


Figure 7.17: Concave mirror with principal axis.

a distance that is half-way between the centre of curvature and the mirror on the principal axis. This point is known as the *focal point* and the distance from the focal point to the mirror is known as the *focal length* (symbol  $f$ ). Since the focal point is the midpoint of the line segment joining the vertex and the center of curvature, the focal length would be one-half the radius of curvature. This fact can come in very handy, remember if you know one then you know the other!



#### Definition: Focal Point

The focal point of a mirror is the midpoint of a line segment joining the vertex and the centre of curvature. It is the position at which all parallel rays are focussed.

Why are we making such a big deal about this point we call the focal point? It has an important property we will use often. A ray parallel to the principal axis hitting the mirror will always be reflected through the focal point. The focal point is the position at which all parallel rays are *focussed*.

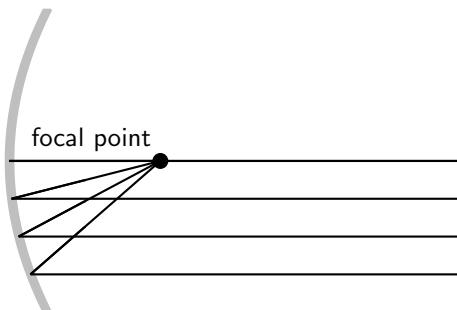


Figure 7.18: All light rays pass through the focal point.

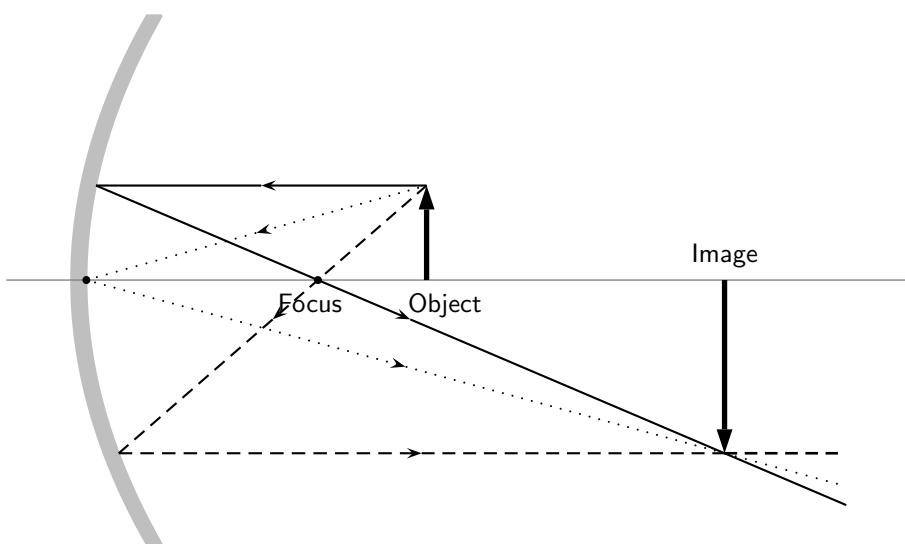


Figure 7.19: A concave mirror with three rays drawn to locate the image. Each incident ray is reflected according to the Law of Reflection. The intersection of the reflected rays gives the location of the image. Here the image is real and inverted.

From Figure 7.19, we see that the image created by a concave mirror is real and inverted, as compared to the virtual and erect image created by a plane mirror.

**Definition: Real Image**

A real image can be cast on a screen; it is inverted, and on the same side of the mirror as the object.

**Extension: Convergence**

A concave mirror is also known as a converging mirror. Light rays appear to converge to the focal point of a concave mirror.

### 7.5.6 Convex Mirrors

The second type of curved mirror we will study are convex mirrors. Convex mirrors have the shape shown in Figure 7.20. As with a plane mirror, the principal axis is a line that is perpendicular to the centre of the mirror.

We have defined the focal point as that point that is half-way along the principal axis between the centre of curvature and the mirror. Now for a convex mirror, this point is *behind* the mirror. A convex mirror has a *negative* focal length because the focal point is behind the mirror.

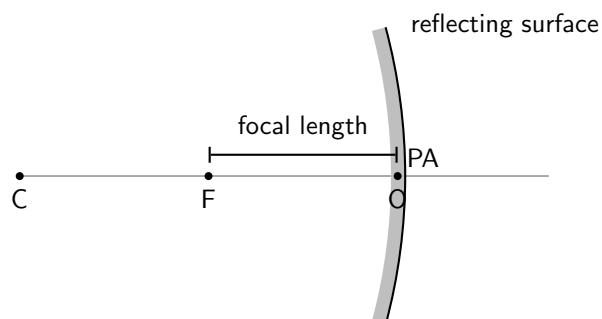


Figure 7.20: Convex mirror with principle axis, focal point (F) and centre of curvature (C). The centre of the mirror is the optical centre (O).

To determine what the image from a convex mirror looks like and where the image is located, we need to remember that a mirror obeys the laws of reflection and that light appears to come from the image. The image created by a convex mirror is shown in Figure 7.21.

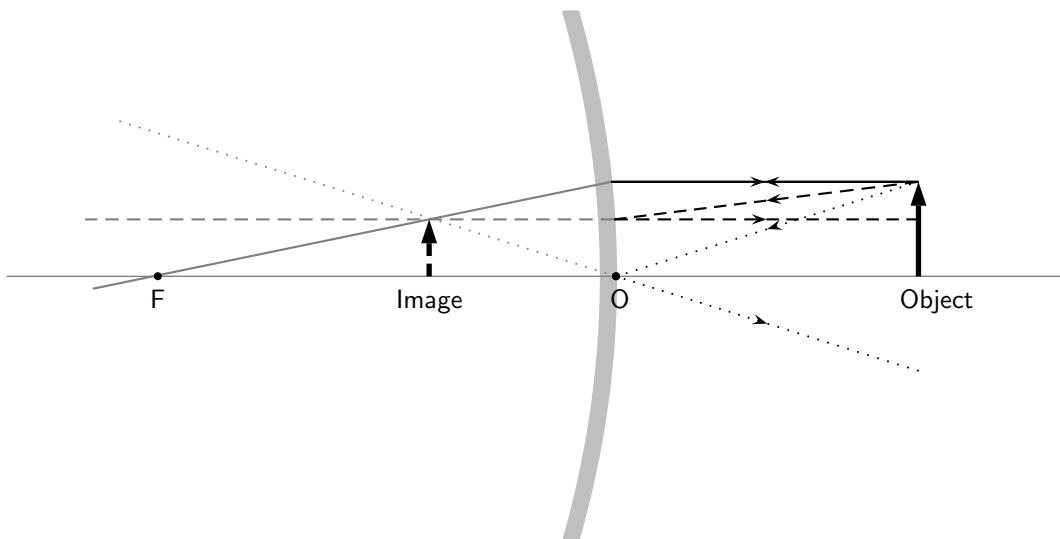


Figure 7.21: A convex mirror with three rays drawn to locate the image. Each incident ray is reflected according to the Law of Reflection. The reflected rays diverge. If the reflected rays are extended behind the mirror, then their intersection gives the location of the image behind the mirror. For a convex mirror, the image is virtual and upright.

From Figure 7.21, we see that the image created by a convex mirror is virtual and upright, as compared to the real and inverted image created by a concave mirror.



#### *Extension: Divergence*

A convex mirror is also known as a diverging mirror. Light rays appear to diverge from the focal point of a convex mirror.

### 7.5.7 Summary of Properties of Mirrors

The properties of mirrors are summarised in Table 7.2.

Table 7.2: Summary of properties of concave and convex mirrors.

Plane	Concave	Convex
-	converging	diverging
virtual image	real image	virtual image
upright	inverted	upright
image behind mirror	image in front of mirror	image behind mirror

### 7.5.8 Magnification

In Figures 7.19 and 7.21, the height of the object and image arrows were different. In any optical system where images are formed from objects, the ratio of the image height,  $h_i$ , to the object height,  $h_o$  is known as the magnification,  $m$ .

$$m = \frac{h_i}{h_o}$$

This is true for the mirror examples we showed above and will also be true for lenses, which will be introduced in the next sections. For a plane mirror, the height of the image is the same as the

height of the object, so the magnification is simply  $m = \frac{h_i}{h_o} = 1$ . If the magnification is greater than 1, the image is larger than the object and is said to be *magnified*. If the magnification is less than 1, the image is smaller than the object so the image is said to be *diminished*.



### Worked Example 36: Magnification

**Question:** A concave mirror forms an image that is 4,8 cm high. The height of the object is 1,6 cm. Calculate the magnification of the mirror.

**Answer**

**Step 1 : Identify what is given and what is asked.**

Image height  $h_i = 4,8$  cm

Object height  $h_o = 1,6$  cm

Magnification  $m = ?$

**Step 2 : Substitute the values and calculate m.**

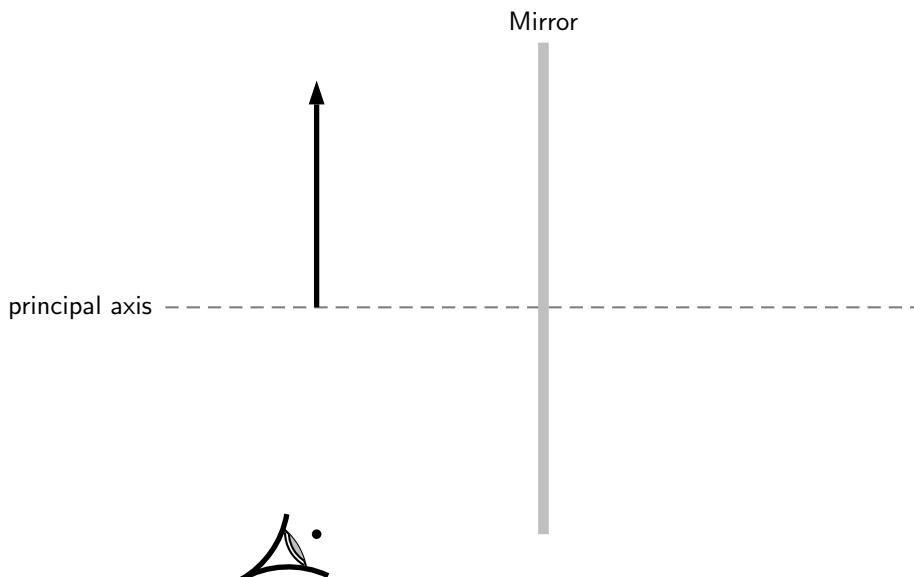
$$\begin{aligned} m &= \frac{h_i}{h_o} \\ &= \frac{4,8}{1,6} \\ &= 3 \end{aligned}$$

The magnification is 3 times.



### Exercise: Mirrors

1. List 5 properties of a virtual image created by reflection from a plane mirror.
2. What angle does the principal axis make with a plane mirror?
3. Is the principal axis a normal to the surface of the plane mirror?
4. Do the reflected rays that contribute to forming the image from a plane mirror obey the law of reflection?
5. If a candle is placed 50 cm in front of a plane mirror, how far behind the plane mirror will the image be? Draw a ray diagram to show how the image is formed.
6. If a stool 0,5 m high is placed 2 m in front of a plane mirror, how far behind the plane mirror will the image be and how high will the image be?
7. If Susan stands 3 m in front of a plane mirror, how far from Susan will her image be located?
8. Explain why ambulances have the word 'ambulance' reversed on the front bonnet of the car?
9. Complete the diagram by filling in the missing lines to locate the image.



10. An object 2 cm high is placed 4 cm in front of a plane mirror. Draw a ray diagram, showing the object, the mirror and the position of the image.
  11. The image of an object is located 5 cm behind a plane mirror. Draw a ray diagram, showing the image, the mirror and the position of the object.
  12. How high must a mirror be so that you can see your whole body in it? Does it make a difference if you change the distance you stand in front of the mirror? Explain.
  13. If 1-year old Tommy crawls towards a mirror at a rate of  $0,3 \text{ m}\cdot\text{s}^{-1}$ , at what speed will Tommy and his image approach each other?
  14. Use a diagram to explain how light converges to the focal point of a concave mirror.
  15. Use a diagram to explain how light diverges away from the focal point of a convex mirror.
  16. An object 1 cm high is placed 4 cm from a concave mirror. If the focal length of the mirror is 2 cm, find the position and size of the image by means of a ray diagram. Is the image real or virtual?
  17. An object 2 cm high is placed 4 cm from a convex mirror. If the focal length of the mirror is 4 cm, find the position and size of the image by means of a ray diagram. Is the image real or virtual?
  18. Calculate the magnification for each of the mirrors in the previous two questions.
- 

## 7.6 Total Internal Reflection and Fibre Optics

### 7.6.1 Total Internal Reflection

---

#### **Activity :: Investigation : Total Internal Reflection**

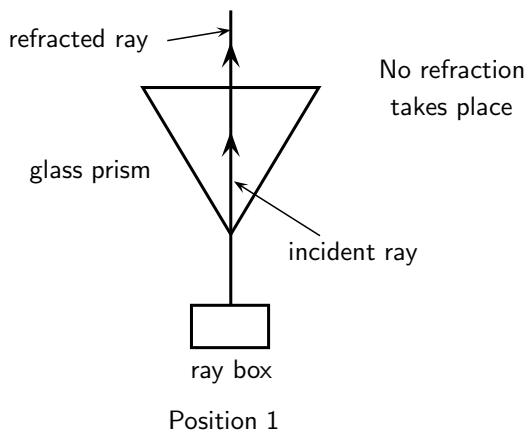
Work in groups of four. Each group will need a raybox (or torch) with slit, triangular glass prism and protractor. If you do not have a raybox, use a torch and stick two pieces of tape over the lens so that only a thin beam of light is visible.

**Aim:**

To investigate total internal reflection.

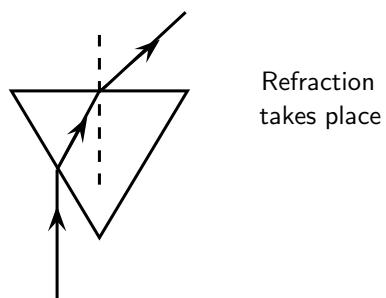
**Method:**

1. Place the raybox next to the glass block so that the light shines right through without any refraction. See "Position 1" in diagram.



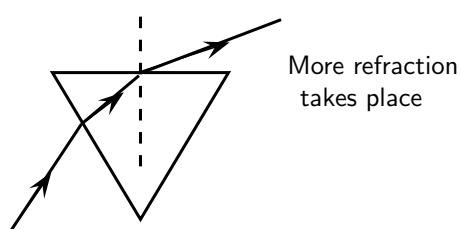
Position 1

2. Move the raybox such that the light is refracted by the glass. See "Position 2".



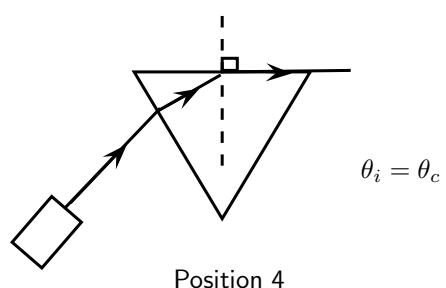
Position 2

3. Move the raybox further and observe what happens.



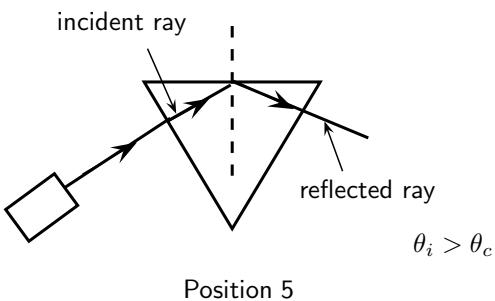
Position 3

4. Move the raybox until the refracted ray seems to disappear. See "Position 4". The angle of the incident light is called the critical angle.



Position 4

5. Move the raybox further and observe what happens. See "Position 5". The light shines back into the glass block. This is called total internal reflection.



When we increase the angle of incidence, we reach a point where the angle of refraction is  $90^\circ$  and the refracted ray runs along the surface of the medium. This angle of incidence is called the critical angle.



#### Definition: Critical Angle

The critical angle is the angle of incidence where the angle of reflection is  $90^\circ$ . The light must shine from a dense to a less dense medium.

If the angle of incidence is bigger than this critical angle, the refracted ray will not emerge from the medium, but will be reflected back into the medium. This is called total internal reflection.

Total internal reflection takes place when

- light shines from an optically denser medium to an optically less dense medium.
- the angle of incidence is greater than the critical angle.



#### Definition: Total Internal Reflection

Total internal reflection takes place when light is reflected back into the medium because the angle of incidence is greater than the critical angle.

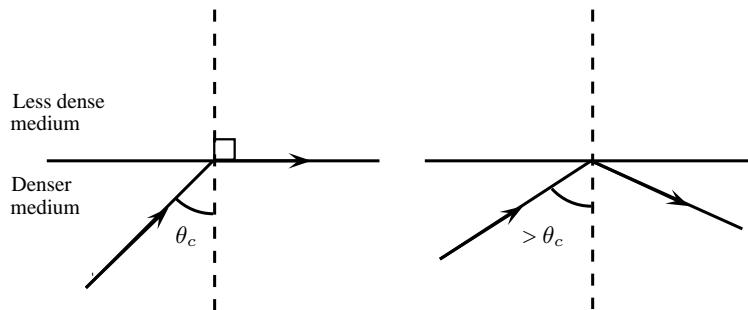


Figure 7.22: Diagrams to show the critical angle and total internal reflection.

Each medium has its own unique critical angle. For example, the critical angle for glass is  $42^\circ$ , and that of water is  $48.8^\circ$ . We can calculate the critical angle for any medium.

#### Calculating the Critical Angle

Now we shall learn how to derive the value of the critical angle for two given media. The process is fairly simple and involves just the use of Snell's Law that we have already studied. To recap, Snell's Law states:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

where  $n_1$  is the refractive index of material 1,  $n_2$  is the refractive index of material 2,  $\theta_1$  is the angle of incidence and  $\theta_2$  is the angle of refraction. For total internal reflection we know that the angle of incidence is the critical angle. So,

$$\theta_1 = \theta_c.$$

However, we also know that the angle of refraction at the critical angle is  $90^\circ$ . So we have:

$$\theta_2 = 90^\circ.$$

We can then write Snell's Law as:

$$n_1 \sin \theta_c = n_2 \sin 90^\circ$$

Solving for  $\theta_c$  gives:

$$\begin{aligned} n_1 \sin \theta_c &= n_2 \sin 90^\circ \\ \sin \theta_c &= \frac{n_2}{n_1} (1) \\ \therefore \theta_c &= \sin^{-1} \left( \frac{n_2}{n_1} \right) \end{aligned}$$



**Important:** Take care that for total internal reflection the incident ray is always in the denser medium.



### Worked Example 37: Critical Angle 1

**Question:** Given that the refractive indices of air and water are 1 and 1,33, respectively, find the critical angle.

**Answer**

#### Step 1 : Determine how to approach the problem

We know that the critical angle is given by:

$$\theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right)$$

#### Step 2 : Solve the problem

$$\begin{aligned} \theta_c &= \sin^{-1} \left( \frac{n_2}{n_1} \right) \\ &= \sin^{-1} \left( \frac{1}{1,33} \right) \\ &= 48,8^\circ \end{aligned}$$

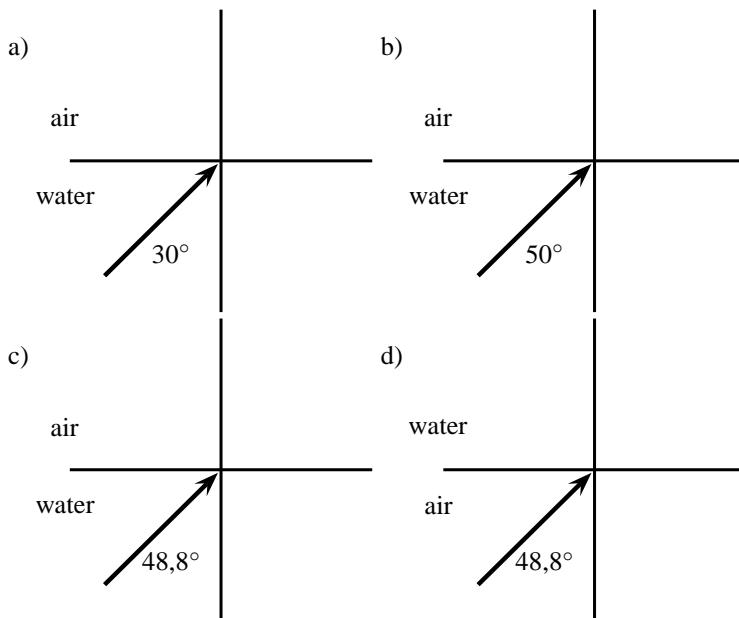
#### Step 3 : Write the final answer

The critical angle for light travelling from water to air is  $48,8^\circ$ .



### Worked Example 38: Critical Angle 2

**Question:** Complete the following ray diagrams to show the path of light in each situation.

**Answer****Step 1 : Identify what is given and what is asked**

The critical angle for water is  $48,8^\circ$ .

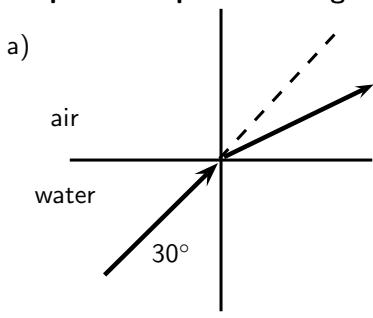
We are asked to complete the diagrams.

For incident angles smaller than  $48,8^\circ$  refraction will occur.

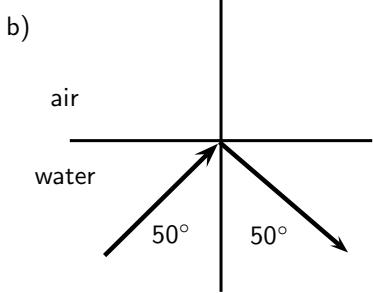
For incident angles greater than  $48,8^\circ$  total internal reflection will occur.

For incident angles equal to  $48,8^\circ$  refraction will occur at  $90^\circ$ .

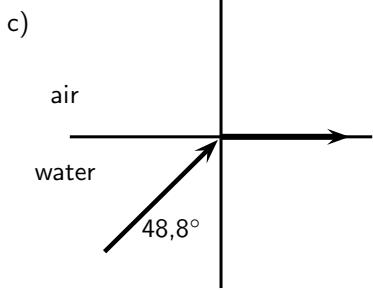
The light must travel from a high optical density to a lower one.

**Step 2 : Complete the diagrams**

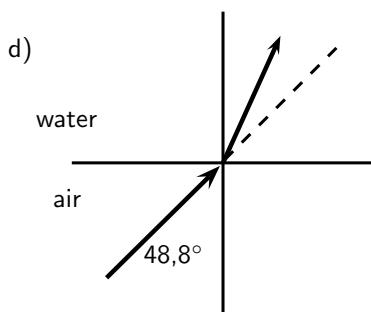
Refraction occurs (ray is bent away from the normal)



Total internal reflection occurs



$$\theta_c = 48,8^\circ$$



Refraction towards the normal (air is less dense than water)

## 7.6.2 Fibre Optics

Total internal reflection is a powerful tool since it can be used to confine light. One of the most common applications of total internal reflection is in *fibre optics*. An optical fibre is a thin, transparent fibre, usually made of glass or plastic, for transmitting light. Optical fibres are usually thinner than a human hair! The construction of a single optical fibre is shown in Figure 7.23.

The basic functional structure of an optical fibre consists of an outer protective *cladding* and an *inner core* through which light pulses travel. The overall diameter of the fibre is about  $125 \mu\text{m}$  ( $125 \times 10^{-6} \text{ m}$ ) and that of the core is just about  $10 \mu\text{m}$  ( $10 \times 10^{-6} \text{ m}$ ). The mode of operation of the optical fibres, as mentioned above, depends on the phenomenon of total internal reflection. The difference in refractive index of the cladding and the core allows total internal reflection in the same way as happens at an air-water surface. If light is incident on a cable end with an angle of incidence greater than the critical angle then the light will remain trapped inside the glass strand. In this way, light travels very quickly down the length of the cable.

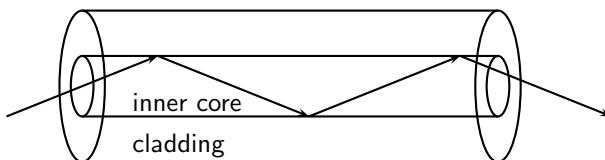


Figure 7.23: Structure of a single optical fibre.

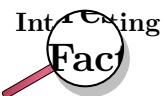
### Fibre Optics in Telecommunications

Optical fibres are most common in telecommunications, because information can be transported over long distances, with minimal loss of data. The minimised loss of data gives optical fibres an advantage over conventional cables.

Data is transmitted from one end of the fibre to another in the form of laser pulses. A single strand is capable of handling over 3000 simultaneous transmissions which is a huge improvement over the conventional co-axial cables. Multiple signal transmission is achieved by sending individual light pulses at slightly different angles. For example if one of the pulses makes a  $72,23^\circ$  angle of incidence then a separate pulse can be sent at an angle of  $72,26^\circ$ ! The transmitted data is received almost instantaneously at the other end of the cable since the information coded onto the laser travels at the speed of light! During transmission over long distances *repeater stations* are used to amplify the signal which has weakened somewhat by the time it reaches the station. The amplified signals are then relayed towards their destination and may encounter several other repeater stations on the way.

### Fibre Optics in Medicine

Optic fibres are used in medicine in *endoscopes*.



Endoscopy means *to look inside* and refers to looking inside the human body for diagnosing medical conditions.

The main part of an endoscope is the optical fibre. Light is shone down the optical fibre and a medical doctor can use the endoscope to look inside a patient. Endoscopes are used to examine the inside of a patient's stomach, by inserting the endoscope down the patient's throat.

Endoscopes allow minimally invasive surgery. This means that a person can be diagnosed and treated through a small incision. This has advantages over open surgery because endoscopy is quicker and cheaper and the patient recovers more quickly. The alternative is open surgery which is expensive, requires more time and is more traumatic for the patient.



### Exercise: Total Internal Reflection and Fibre Optics

1. Describe total internal reflection, referring to the conditions that must be satisfied for total internal reflection to occur.
2. Define what is meant by the *critical angle* when referring to total internal reflection. Include a ray diagram to explain the concept.
3. Will light travelling from diamond to silicon ever undergo total internal reflection?
4. Will light travelling from sapphire to diamond undergo total internal reflection?
5. What is the critical angle for light traveling from air to acetone?
6. Light traveling from diamond to water strikes the interface with an angle of incidence of  $86^\circ$ . Calculate the critical angle to determine whether the light be totally internally reflected and so be trapped within the water.
7. Which of the following interfaces will have the largest critical angle?
  - (a) a glass to water interface
  - (b) a diamond to water interface
  - (c) a diamond to glass interface
8. If the fibre optic strand is made from glass, determine the critical angle of the light ray so that the ray stays within the fibre optic strand.
9. A glass slab is inserted in a tank of water. If the refractive index of water is 1,33 and that of glass is 1,5, find the critical angle.
10. A diamond ring is placed in a container full of glycerin. If the critical angle is found to be  $37,4^\circ$  and the refractive index of glycerin is given to be 1,47, find the refractive index of diamond.
11. An optical fibre is made up of a core of refractive index 1,9, while the refractive index of the cladding is 1,5. Calculate the maximum angle which a light pulse can make with the wall of the core. NOTE: The question does not ask for the angle of incidence but for the angle made by the ray with the wall of the core, which will be equal to  $90^\circ$ - angle of incidence.

## 7.7 Summary

1. We can see objects when light from the objects enters our eyes.
2. Light rays are thin imaginary lines of light and are indicated in drawings by means of arrows.
3. Light travels in straight lines. Light can therefore not travel around corners. Shadows are formed because light shines in straight lines.
4. Light rays reflect off surfaces. The incident ray shines on the surface and the reflected ray is the one that bounces off the surface. The surface normal is the perpendicular line to the surface where the light strikes the surface.
5. The angle of incidence is the angle between the incident ray and the surface, and the angle of reflection is the angle between the reflected ray and the surface.
6. The Law of Reflection states the angle of incidence is equal to the angle of reflection and that the reflected ray lies in the plane of incidence.
7. Specular reflection takes place when parallel rays fall on a surface and they leave the object as parallel rays. Diffuse reflection takes place when parallel rays are reflected in different directions.
8. Refraction is the bending of light when it travels from one medium to another. Light travels at different speeds in different media.

9. The refractive index of a medium is a measure of how easily light travels through the medium. It is a ratio of the speed of light in a vacuum to the speed of light in the medium.

$$n = \frac{c}{v}$$

10. Snell's Law gives the relationship between the refractive indices, angles of incidence and reflection of two media.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

11. Light travelling from one medium to another of lighter optical density will be refracted towards the normal.

Light travelling from one medium to another of lower optical density will be refracted away from the normal.

12. Objects in a medium (e.g. under water) appear closer to the surface than they really are. This is due to the refraction of light, and the refractive index of the medium.

$$n = \frac{\text{real depth}}{\text{apparent depth}}$$

13. Mirrors are highly reflective surfaces. Flat mirrors are called plane mirrors. Curved mirrors can be convex or concave. The properties of the images formed by mirrors are summarised in Table 3.2.

14. A real image can be cast on a screen, is inverted and in front of the mirror.  
A virtual image cannot be cast on a screen, is upright and behind the mirror.

15. The magnification of a mirror is how many times the image is bigger or smaller than the object.

$$m = \frac{\text{image height } (h_i)}{\text{object height } (h_o)}$$

16. The critical angle of a medium is the angle of incidence when the angle of refraction is  $90^\circ$  and the refracted ray runs along the interface between the two media.

17. Total internal reflection takes place when light travels from one medium to another of lower optical density. If the angle of incidence is greater than the critical angle for the medium, the light will be reflected back into the medium. No refraction takes place.

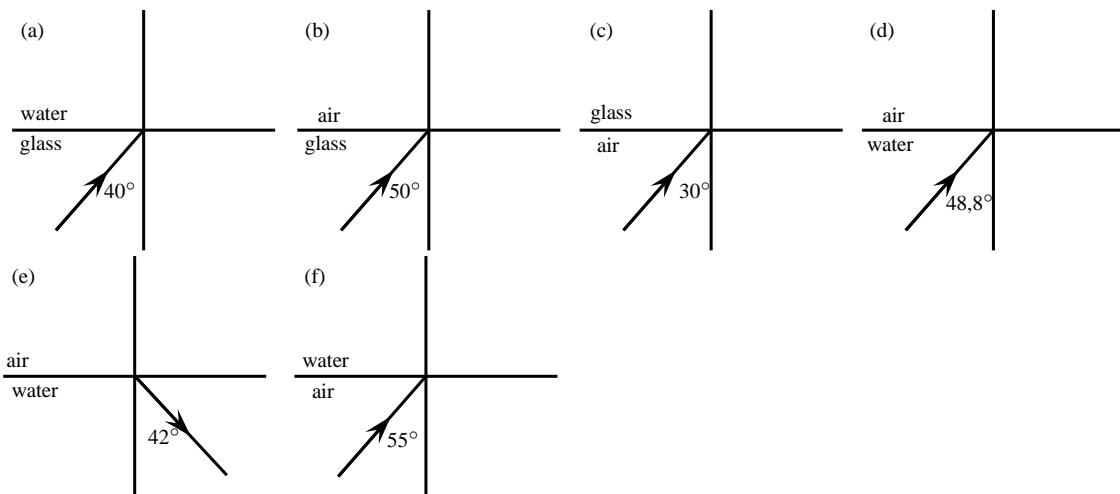
18. Total internal reflection is used in optical fibres in telecommunication and in medicine in endoscopes. Optical fibres transmit information much more quickly and accurately than traditional methods.

## 7.8 Exercises

1. Give one word for each of the following descriptions:
  - 1.1 The image that is formed by a plane mirror.
  - 1.2 The perpendicular line that is drawn at right angles to a reflecting surface at the point of incidence.
  - 1.3 The bending of light as it travels from one medium to another.
  - 1.4 The ray of light that falls in on an object.
  - 1.5 A type of mirror that focuses all rays behind the mirror.
2. State whether the following statements are TRUE or FALSE. If they are false, rewrite the statement correcting it.
  - 2.1 The refractive index of a medium is an indication of how fast light will travel through the medium.
  - 2.2 Total internal refraction takes place when the incident angle is larger than the critical angle.
  - 2.3 The magnification of an object can be calculated if the speed of light in a vacuum and the speed of light in the medium is known.
  - 2.4 The speed of light in a vacuum is about  $3 \times 10^8 \text{ m.s}^{-1}$ .
  - 2.5 Specular reflection takes place when light is reflected off a rough surface.
3. Choose words from Column B to match the concept/description in Column A. All the appropriate words should be identified. Words can be used more than once.

	<u>Column A</u>	<u>Column B</u>
(a)	Real image	Upright
(b)	Virtual image	Can be cast on a screen
(c)	Concave mirror	In front
(d)	Convex mirror	Behind
(e)	Plane mirror	Inverted Light travels to it Upside down Light does not reach it Erect Same size

4. Complete the following ray diagrams to show the path of light.



5. A ray of light strikes a surface at  $35^\circ$  to the surface normal. Draw a ray diagram showing the incident ray, reflected ray and surface normal. Calculate the angles of incidence and reflection and fill them in on your diagram.
6. Light travels from glass ( $n = 1,5$ ) to acetone ( $n = 1,36$ ). The angle of incidence is  $25^\circ$ .

- 6.1 Describe the path of light as it moves into the acetone.
- 6.2 Calculate the angle of refraction.
- 6.3 What happens to the speed of the light as it moves from the glass to the acetone?
- 6.4 What happens to the wavelength of the light as it moves into the acetone?
- 6.5 What is the name of the phenomenon that occurs at the interface between the two media?
7. A stone lies at the bottom of a swimming pool. The water is 120 cm deep. The refractive index of water is 1,33. How deep does the stone appear to be?
8. Light strikes the interface between air and an unknown medium with an incident angle of  $32^\circ$ . The angle of refraction is measured to be  $48^\circ$ . Calculate the refractive index of the medium and identify the medium.
9. Explain what total internal reflection is and how it is used in medicine and telecommunications. Why is this technology much better to use?
10. A candle 10 cm high is placed 25 cm in front of a plane mirror. Draw a ray diagram to show how the image is formed. Include all labels and write down the properties of the image.
11. A virtual image, 4 cm high, is formed 3 cm from a plane mirror. Draw a labelled ray diagram to show the position and height of the object. What is the magnification?
12. An object, 3 cm high, is placed 4 cm from a concave mirror of focal length 2 cm. Draw a labelled ray diagram to find the position, height and properties of the image.
13. An object, 2 cm high, is placed 3 cm from a convex mirror. The magnification is 0,5. Calculate the focal length of the mirror.



## **Appendix A**

# **GNU Free Documentation License**

Version 1.2, November 2002

Copyright © 2000,2001,2002 Free Software Foundation, Inc.

59 Temple Place, Suite 330, Boston, MA 02111-1307 USA

Everyone is permitted to copy and distribute verbatim copies of this license document, but changing it is not allowed.

### **PREAMBLE**

The purpose of this License is to make a manual, textbook, or other functional and useful document "free" in the sense of freedom: to assure everyone the effective freedom to copy and redistribute it, with or without modifying it, either commercially or non-commercially. Secondary, this License preserves for the author and publisher a way to get credit for their work, while not being considered responsible for modifications made by others.

This License is a kind of "copyleft", which means that derivative works of the document must themselves be free in the same sense. It complements the GNU General Public License, which is a copyleft license designed for free software.

We have designed this License in order to use it for manuals for free software, because free software needs free documentation: a free program should come with manuals providing the same freedoms that the software does. But this License is not limited to software manuals; it can be used for any textual work, regardless of subject matter or whether it is published as a printed book. We recommend this License principally for works whose purpose is instruction or reference.

### **APPLICABILITY AND DEFINITIONS**

This License applies to any manual or other work, in any medium, that contains a notice placed by the copyright holder saying it can be distributed under the terms of this License. Such a notice grants a world-wide, royalty-free license, unlimited in duration, to use that work under the conditions stated herein. The "Document", below, refers to any such manual or work. Any member of the public is a licensee, and is addressed as "you". You accept the license if you copy, modify or distribute the work in a way requiring permission under copyright law.

A "Modified Version" of the Document means any work containing the Document or a portion of it, either copied verbatim, or with modifications and/or translated into another language.

A "Secondary Section" is a named appendix or a front-matter section of the Document that deals exclusively with the relationship of the publishers or authors of the Document to the Document's overall subject (or to related matters) and contains nothing that could fall directly within that overall subject. (Thus, if the Document is in part a textbook of mathematics, a

Secondary Section may not explain any mathematics.) The relationship could be a matter of historical connection with the subject or with related matters, or of legal, commercial, philosophical, ethical or political position regarding them.

The “Invariant Sections” are certain Secondary Sections whose titles are designated, as being those of Invariant Sections, in the notice that says that the Document is released under this License. If a section does not fit the above definition of Secondary then it is not allowed to be designated as Invariant. The Document may contain zero Invariant Sections. If the Document does not identify any Invariant Sections then there are none.

The “Cover Texts” are certain short passages of text that are listed, as Front-Cover Texts or Back-Cover Texts, in the notice that says that the Document is released under this License. A Front-Cover Text may be at most 5 words, and a Back-Cover Text may be at most 25 words.

A “Transparent” copy of the Document means a machine-readable copy, represented in a format whose specification is available to the general public, that is suitable for revising the document straightforwardly with generic text editors or (for images composed of pixels) generic paint programs or (for drawings) some widely available drawing editor, and that is suitable for input to text formatters or for automatic translation to a variety of formats suitable for input to text formatters. A copy made in an otherwise Transparent file format whose markup, or absence of markup, has been arranged to thwart or discourage subsequent modification by readers is not Transparent. An image format is not Transparent if used for any substantial amount of text. A copy that is not “Transparent” is called “Opaque”.

Examples of suitable formats for Transparent copies include plain ASCII without markup, Texinfo input format, L<sup>A</sup>T<sub>E</sub>X input format, SGML or XML using a publicly available DTD and standard-conforming simple HTML, PostScript or PDF designed for human modification. Examples of transparent image formats include PNG, XCF and JPG. Opaque formats include proprietary formats that can be read and edited only by proprietary word processors, SGML or XML for which the DTD and/or processing tools are not generally available, and the machine-generated HTML, PostScript or PDF produced by some word processors for output purposes only.

The “Title Page” means, for a printed book, the title page itself, plus such following pages as are needed to hold, legibly, the material this License requires to appear in the title page. For works in formats which do not have any title page as such, “Title Page” means the text near the most prominent appearance of the work’s title, preceding the beginning of the body of the text.

A section “Entitled XYZ” means a named subunit of the Document whose title either is precisely XYZ or contains XYZ in parentheses following text that translates XYZ in another language. (Here XYZ stands for a specific section name mentioned below, such as “Acknowledgements”, “Dedications”, “Endorsements”, or “History”.) To “Preserve the Title” of such a section when you modify the Document means that it remains a section “Entitled XYZ” according to this definition.

The Document may include Warranty Disclaimers next to the notice which states that this License applies to the Document. These Warranty Disclaimers are considered to be included by reference in this License, but only as regards disclaiming warranties: any other implication that these Warranty Disclaimers may have is void and has no effect on the meaning of this License.

## VERBATIM COPYING

You may copy and distribute the Document in any medium, either commercially or non-commercially, provided that this License, the copyright notices, and the license notice saying this License applies to the Document are reproduced in all copies, and that you add no other conditions whatsoever to those of this License. You may not use technical measures to obstruct or control the reading or further copying of the copies you make or distribute. However, you may accept compensation in exchange for copies. If you distribute a large enough number of copies you must also follow the conditions in section A.

You may also lend copies, under the same conditions stated above, and you may publicly display copies.

## **COPYING IN QUANTITY**

If you publish printed copies (or copies in media that commonly have printed covers) of the Document, numbering more than 100, and the Document's license notice requires Cover Texts, you must enclose the copies in covers that carry, clearly and legibly, all these Cover Texts: Front-Cover Texts on the front cover, and Back-Cover Texts on the back cover. Both covers must also clearly and legibly identify you as the publisher of these copies. The front cover must present the full title with all words of the title equally prominent and visible. You may add other material on the covers in addition. Copying with changes limited to the covers, as long as they preserve the title of the Document and satisfy these conditions, can be treated as verbatim copying in other respects.

If the required texts for either cover are too voluminous to fit legibly, you should put the first ones listed (as many as fit reasonably) on the actual cover, and continue the rest onto adjacent pages.

If you publish or distribute Opaque copies of the Document numbering more than 100, you must either include a machine-readable Transparent copy along with each Opaque copy, or state in or with each Opaque copy a computer-network location from which the general network-using public has access to download using public-standard network protocols a complete Transparent copy of the Document, free of added material. If you use the latter option, you must take reasonably prudent steps, when you begin distribution of Opaque copies in quantity, to ensure that this Transparent copy will remain thus accessible at the stated location until at least one year after the last time you distribute an Opaque copy (directly or through your agents or retailers) of that edition to the public.

It is requested, but not required, that you contact the authors of the Document well before redistributing any large number of copies, to give them a chance to provide you with an updated version of the Document.

## **MODIFICATIONS**

You may copy and distribute a Modified Version of the Document under the conditions of sections A and A above, provided that you release the Modified Version under precisely this License, with the Modified Version filling the role of the Document, thus licensing distribution and modification of the Modified Version to whoever possesses a copy of it. In addition, you must do these things in the Modified Version:

1. Use in the Title Page (and on the covers, if any) a title distinct from that of the Document, and from those of previous versions (which should, if there were any, be listed in the History section of the Document). You may use the same title as a previous version if the original publisher of that version gives permission.
2. List on the Title Page, as authors, one or more persons or entities responsible for authorship of the modifications in the Modified Version, together with at least five of the principal authors of the Document (all of its principal authors, if it has fewer than five), unless they release you from this requirement.
3. State on the Title page the name of the publisher of the Modified Version, as the publisher.
4. Preserve all the copyright notices of the Document.
5. Add an appropriate copyright notice for your modifications adjacent to the other copyright notices.

6. Include, immediately after the copyright notices, a license notice giving the public permission to use the Modified Version under the terms of this License, in the form shown in the Addendum below.
7. Preserve in that license notice the full lists of Invariant Sections and required Cover Texts given in the Document's license notice.
8. Include an unaltered copy of this License.
9. Preserve the section Entitled "History", Preserve its Title, and add to it an item stating at least the title, year, new authors, and publisher of the Modified Version as given on the Title Page. If there is no section Entitled "History" in the Document, create one stating the title, year, authors, and publisher of the Document as given on its Title Page, then add an item describing the Modified Version as stated in the previous sentence.
10. Preserve the network location, if any, given in the Document for public access to a Transparent copy of the Document, and likewise the network locations given in the Document for previous versions it was based on. These may be placed in the "History" section. You may omit a network location for a work that was published at least four years before the Document itself, or if the original publisher of the version it refers to gives permission.
11. For any section Entitled "Acknowledgements" or "Dedications", Preserve the Title of the section, and preserve in the section all the substance and tone of each of the contributor acknowledgements and/or dedications given therein.
12. Preserve all the Invariant Sections of the Document, unaltered in their text and in their titles. Section numbers or the equivalent are not considered part of the section titles.
13. Delete any section Entitled "Endorsements". Such a section may not be included in the Modified Version.
14. Do not re-title any existing section to be Entitled "Endorsements" or to conflict in title with any Invariant Section.
15. Preserve any Warranty Disclaimers.

If the Modified Version includes new front-matter sections or appendices that qualify as Secondary Sections and contain no material copied from the Document, you may at your option designate some or all of these sections as invariant. To do this, add their titles to the list of Invariant Sections in the Modified Version's license notice. These titles must be distinct from any other section titles.

You may add a section Entitled "Endorsements", provided it contains nothing but endorsements of your Modified Version by various parties—for example, statements of peer review or that the text has been approved by an organisation as the authoritative definition of a standard.

You may add a passage of up to five words as a Front-Cover Text, and a passage of up to 25 words as a Back-Cover Text, to the end of the list of Cover Texts in the Modified Version. Only one passage of Front-Cover Text and one of Back-Cover Text may be added by (or through arrangements made by) any one entity. If the Document already includes a cover text for the same cover, previously added by you or by arrangement made by the same entity you are acting on behalf of, you may not add another; but you may replace the old one, on explicit permission from the previous publisher that added the old one.

The author(s) and publisher(s) of the Document do not by this License give permission to use their names for publicity for or to assert or imply endorsement of any Modified Version.

## COMBINING DOCUMENTS

You may combine the Document with other documents released under this License, under the terms defined in section A above for modified versions, provided that you include in the

combination all of the Invariant Sections of all of the original documents, unmodified, and list them all as Invariant Sections of your combined work in its license notice, and that you preserve all their Warranty Disclaimers.

The combined work need only contain one copy of this License, and multiple identical Invariant Sections may be replaced with a single copy. If there are multiple Invariant Sections with the same name but different contents, make the title of each such section unique by adding at the end of it, in parentheses, the name of the original author or publisher of that section if known, or else a unique number. Make the same adjustment to the section titles in the list of Invariant Sections in the license notice of the combined work.

In the combination, you must combine any sections Entitled "History" in the various original documents, forming one section Entitled "History"; likewise combine any sections Entitled "Acknowledgements", and any sections Entitled "Dedications". You must delete all sections Entitled "Endorsements".

## **COLLECTIONS OF DOCUMENTS**

You may make a collection consisting of the Document and other documents released under this License, and replace the individual copies of this License in the various documents with a single copy that is included in the collection, provided that you follow the rules of this License for verbatim copying of each of the documents in all other respects.

You may extract a single document from such a collection, and distribute it individually under this License, provided you insert a copy of this License into the extracted document, and follow this License in all other respects regarding verbatim copying of that document.

## **AGGREGATION WITH INDEPENDENT WORKS**

A compilation of the Document or its derivatives with other separate and independent documents or works, in or on a volume of a storage or distribution medium, is called an "aggregate" if the copyright resulting from the compilation is not used to limit the legal rights of the compilation's users beyond what the individual works permit. When the Document is included in an aggregate, this License does not apply to the other works in the aggregate which are not themselves derivative works of the Document.

If the Cover Text requirement of section A is applicable to these copies of the Document, then if the Document is less than one half of the entire aggregate, the Document's Cover Texts may be placed on covers that bracket the Document within the aggregate, or the electronic equivalent of covers if the Document is in electronic form. Otherwise they must appear on printed covers that bracket the whole aggregate.

## **TRANSLATION**

Translation is considered a kind of modification, so you may distribute translations of the Document under the terms of section A. Replacing Invariant Sections with translations requires special permission from their copyright holders, but you may include translations of some or all Invariant Sections in addition to the original versions of these Invariant Sections. You may include a translation of this License, and all the license notices in the Document, and any Warranty Disclaimers, provided that you also include the original English version of this License and the original versions of those notices and disclaimers. In case of a disagreement between the translation and the original version of this License or a notice or disclaimer, the original version will prevail.

If a section in the Document is Entitled "Acknowledgements", "Dedications", or "History", the requirement (section A) to Preserve its Title (section A) will typically require changing the

actual title.

## TERMINATION

You may not copy, modify, sub-license, or distribute the Document except as expressly provided for under this License. Any other attempt to copy, modify, sub-license or distribute the Document is void, and will automatically terminate your rights under this License. However, parties who have received copies, or rights, from you under this License will not have their licenses terminated so long as such parties remain in full compliance.

## FUTURE REVISIONS OF THIS LICENSE

The Free Software Foundation may publish new, revised versions of the GNU Free Documentation License from time to time. Such new versions will be similar in spirit to the present version, but may differ in detail to address new problems or concerns. See <http://www.gnu.org/copyleft/>.

Each version of the License is given a distinguishing version number. If the Document specifies that a particular numbered version of this License "or any later version" applies to it, you have the option of following the terms and conditions either of that specified version or of any later version that has been published (not as a draft) by the Free Software Foundation. If the Document does not specify a version number of this License, you may choose any version ever published (not as a draft) by the Free Software Foundation.

## ADDENDUM: How to use this License for your documents

To use this License in a document you have written, include a copy of the License in the document and put the following copyright and license notices just after the title page:

Copyright © YEAR YOUR NAME. Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

If you have Invariant Sections, Front-Cover Texts and Back-Cover Texts, replace the "with...Texts." line with this:

with the Invariant Sections being LIST THEIR TITLES, with the Front-Cover Texts being LIST, and with the Back-Cover Texts being LIST.

If you have Invariant Sections without Cover Texts, or some other combination of the three, merge those two alternatives to suit the situation.

If your document contains nontrivial examples of program code, we recommend releasing these examples in parallel under your choice of free software license, such as the GNU General Public License, to permit their use in free software.