

# Package ‘multisensi’

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**Type** Package

**Title** Multivariate Sensitivity Analysis

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**Description** An R library for performing sensitivity analysis on a model with multivariate output

**License** CeCILL-2

**Repository** CRAN

**LazyLoad** yes

**Depends** R (>= 2.8.0)

**Suggests** MASS

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multisensi-package *Multivariate sensitivity Analysis*

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## Description

Sensitivity Analysis (SA) for models with multivariate output

## Details

This package contains three methods for performing sensitivity analysis on simulation models with multivariate output:

i) `gsi` function for the Generalised Sensitivity Analysis (Lamboni et al., 2009) based on inertia decomposition. This method synthesizes the information that is spread between the time outputs or between the principal components and produces a unique sensitivity index for each factor.

ii) `gsi` function for the componentwise sensitivity analysis obtained by computing sensitivity indices on principal components (Campbell et al., 2006)

iii) `dynsi` function for the dynamic sensitivity analysis obtained by computing sensitivity indices on each output variable.

For all three methods, sensitivity indices are calculated presently by using a factorial design and a classical ANOVA decomposition.

### Simulation model management

The multisensi package works on simulation models coded either in **R** or using an external language (typically as an executable file). Models coded in **R** must be either functions or objects that have a predict method, such as `lm` objects. Models defined as functions will be called once with an expression of the form  $y \leftarrow f(X)$  where  $X$  is a vector containing a combination of levels of the input factors, and  $y$  is the output vector of length  $q$ , where  $q$  is the number of output variables. If the model is external to **R**, for instance a computational code, it must be analyzed with the decoupled approach: the methods require an input data frame ( $X$ ) containing all the combinations of the input levels and the outputs data frame ( $Y$ ) containing the response of the model corresponding to these combinations. The size of  $X$  is  $n * p$  and the size of  $Y$  is  $n * q$  where  $p$  is the number of the input factor,  $q$  is the number of the model outputs and  $n$  is the number of all the combinations of the input levels. This approach can also be used on **R** models that do not fit the required specifications.

## Author(s)

Matieyendou Lamboni <clamboni@yahoo.fr>, Hervé Monod <herve.monod@jouy.inra.fr>

## References

- Lamboni, M., Makowski, D., Monod, H., 2009. Multivariate global sensitivity analysis for dynamic crop models. *Field Crops Research*, volume 113. pp. 312-320
- Lamboni, M., Makowski, D., Monod, H., 2009. Multivariate sensitivity analysis to measure global contribution of input factors in dynamic models (submitted in october 2009 to *Reliability Engineering \& System Safety*)
- Saltelli, A., Chan, K., Scott, E.M. eds, 2000. *Sensitivity Analysis* Wiley, New York.

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acp

*Principal Component Analysis (PCA)*

---

## Description

Principal Component Analysis (PCA) for the generalized sensitivity analysis

## Usage

```
ACP(simuls, normalized = TRUE)
```

## Arguments

<code>simuls</code>	data frame, typically a multivariate model output
<code>normalized</code>	If TRUE, a normalized PCA is performed

## Value

an object of class `prcomp`; see the `prcomp` help for further details

## Note

This is essentially an internal function for the `multisensi` package

## Author(s)

M. LAMBONI

## See Also

`prcomp`

---

anoasg

*Dynamic main and total sensitivity indices*


---

### Description

Function to compute the main and total dynamic sensitivity indices

### Usage

```
anoasg(ANO, nbcomp = 2)
```

### Arguments

ANO	anova object obtained from the anovadec function
nbcomp	number of model output to be considered

### Value

A list with the following components:

**SI** data frame of first order, second order, ... indices

**tSI** data frame of total sensitivity

**mSI** data frame of main sensitivity indices

**iSI** data frame of interaction sensitivity indices

**indic.fact** data frame of anova object attribute

### Note

This is essentially an internal function for the multisensi package

### Author(s)

M. LAMBONI

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anovadec

*Computation of several anovas on the output of a PCA*


---

### Description

A function to compute sum of squares decomposition on principal components by using the aov function

### Usage

```
anovadec(ACP, plan, ord.inter, nbcomp = 2)
```

**Arguments**

ACP	data.frame of model output, usually the x component of a PCA object
plan	data.frame of input design
ord.inter	ANOVA formula like "A+B+C+A:B" OR an integer giving the maximum interaction order (1 for main effects)
nbcomp	number of principal components to be considered (e.g 3)

**Value**

The anovadec function returns a two-component list:

**aov** list of AOV objects

**PC** prediction of output

**Note**

This is essentially an internal function for the multisensi package

**Author(s)**

M. LAMBONI

**See Also**

aov

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asg

---

*Main and total generalized sensitivity computation*


---

**Description**

Function to compute the main and total generalized sensitivity indices

**Usage**

```
asg(ANO, ACP, sigma.car, nbcomp = 2)
```

**Arguments**

ANO	ANOVA Object obtained from anovadec function
ACP	ACP object
sigma.car	Inertia for the model output
nbcomp	Numbers of principal component (PC) to be considered

**Value**

A list with the following components:

**SI** data frame of first order, second order, ... indices

**mSI** data frame of main sensitivity indices

**tSI** data frame of total sensitivity

**iSI** data frame of interaction sensitivity indices

**cor** data frame of correlation between PCs and model output

**inertia** Vector of Inertia explained by PCs

**indic.fact** data frame of anova object attribute

**Note**

This is essentially an internal function for the multisensi package

**Author(s)**

M. LAMBONI

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biomasse

*The Winter Wheat Dynamic Model*

---

**Description**

The Winter Wheat Dynamic Model, a toy model to illustrate the main multisensi methods

**Usage**

```
biomasse(input, climdata, annee = 3)
```

**Arguments**

**input** vector of input value

**annee** year

**climdata** a meteorological data.frame specific to biomasse

**Details**

The Winter Wheat Dry Matter model (WWDm) is a dynamic crop model running at a daily time step (Makowski et al, 2004). It has two state variables, the above-ground winter wheat dry matter  $U(t)$ , in  $g/m^2$  and the leaf area index  $LAI(t)$  with  $t$  the day number from sowing ( $t = 1$ ) to harvest ( $t = 223$ ). In the multisensi-package implementation, the `biomasse` function simulates the output for only one parameter set (the first row of `input` if it is a matrix or a data.frame).

**Value**

a vector of daily dry matter increase of the Winter Wheat biomass, over 223 days

**Author(s)**

initially Makowski, D., 2004

**References**

Makowski, D., Jeuffroy, M.-H., Gu\'erif, M., 2004 Bayesian methods for updating crop model predictions, applications for predicting biomass and grain protein content. In: Bayesian Statistics and Quality Modelling in the Agro-Food Production Chain (van Boekel et al. eds), pp. 57-68. Kluwer, Dordrecht

Monod, H., Naud, C., Makowski, D., 2006 Uncertainty and sensitivity analysis for crop models. In: Working with Dynamic Crop Models (Wallach D., Makowski D. and Jones J. eds), pp. 55-100. Elsevier, Amsterdam

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Climat

*Climate data*

---

**Description**

Climate data for the WWDM model (needed by the `biomasse` function)

**Usage**

```
data(Climat)
```

**Format**

A data frame with 3126 observations on the following 4 variables.

`ANNEE` a factor with levels 1 to 14, indicating 14 different years

`RG` daily radiation variable

`Tmin` daily maximum temperature

`Tmax` daily minimum temperature

**Source**

Makowski, D., Jeuffroy, M.-H., Gu\'erif, M., 2004 Bayesian methods for updating crop model predictions, applications for predicting biomass and grain protein content. In: Bayesian Statistics and Quality Modelling in the Agro-Food Production Chain (van Boekel et al. eds), pp. 57-68. Kluwer, Dordrecht.

Monod, H., Naud, C., Makowski, D., 2006 Uncertainty and sensitivity analysis for crop models. In: Working with Dynamic Crop Models (Wallach D., Makowski D. and Jones J. eds), pp. 55-100. Elsevier, Amsterdam

dynsi

*Dynamic Sensitivity Indices: DSI***Description**

dynsi implements the Dynamic Sensitivity Indices. This method allows to compute classical Sensitivity Indices on each output variable of a dynamic or multivariate model by using the ANOVA decomposition

**Usage**

```
dynsi(formula, model, factors, cumul = FALSE, simulonly=FALSE,
      nb.outp = NULL, Name.File=NULL, ...)
```

**Arguments**

formula	ANOVA formula like "A+B+c+A:B" OR an integer equal to the maximum interaction order in the sensitivity model
model	output data.frame OR the name of the R-function which calculates the model output. The only argument of this function must be a vector containing the input factors values
factors	input data.frame (the design) if model is a data.frame OR a list of factors levels such as <code>factor.example &lt;- list(A=c(0,1), B=c(0,1,4))</code>
cumul	logical value. If TRUE the sensitivity analysis will be done on the cumulative outputs
simulonly	logical value. If TRUE the program stops after calculating the design and the model outputs
nb.outp	The first nb.outp number of model outputs to be considered. If NULL all the outputs are considered
Name.File	optional name of a R script file containing the R-function that calculates the simulation model. e.g "exc.ssc"
...	possible fixed parameters of the model function

**Details**

If factors is a list of factors, the dynsi function generates a complete factorial design. If it is a data.frame, dynsi expects that each column is associated with an input factor.

**Value**

dynsi returns a list of class "dynsi", containing all the input arguments detailed before, plus the following components:

X	a data.frame containing the experimental design (input samples)
Y	a data.frame containing the output matrix (response)
SI	a data.frame containing the Sensitivity Indices (SI) on each output variable of the model
mSI	a data.frame of principal SI on each output variable



tSI                    a data.frame containing the total SI on each output variable  
 iSI                    a data.frame of interaction SI on each output variable  
 ...

### Author(s)

Matieyendou LAMBONI

### References

M. Lamboni, D. Makowski and H. Monod, 2009. Multivariate global sensitivity analysis for dynamic crop models. *Field Crops Research*, 113, 312-320.  
 A. Saltelli, K. Chan and E. M. Scott eds, 2000. *Sensitivity Analysis*. Wiley, New York.

### See Also

gsi

### Examples

```
##---- Should be DIRECTLY executable !! ----
# Test case : the Winter Wheat Dynamic Models (WWDM)
# input factors design,
data(plan)
# input Climate variables
data(Climat)
DYNSI <- dynsi(2, biomasse, plan, nb.outp=10, climdata=Climat)
summary(DYNSI)
print(DYNSI)
plot(DYNSI)
#graph.bar(DYNSI,col=1, beside=F) # sensitivity bar plot
#                                # for the first output (col=1)
#graph.bar(DYNSI,col=2, xmax=1)  #
```

---

graph.bar

*Sensitivity index bar plot*


---

### Description

A function that plots sensitivity indices by a bar graph

### Usage

```
graph.bar(x, col = 1, nb.plot = 15, xmax = NULL, beside = TRUE, ...)
```

**Arguments**

<code>x</code>	an object of class <code>gsi</code> or <code>dynsi</code>
<code>col</code>	the column number of GSI to represent in the bar graph
<code>nb.plot</code>	number of input factors to be considered
<code>xmax</code>	a user-defined maximal $x$ value ( $x \leq 1$ ) in all the bar graphs that show sensitivity indices; or <code>NULL</code> if the user wants to keep default values
<code>beside</code>	if <code>TRUE</code> , the main and total sensitivity indices are represented by two bars; if <code>FALSE</code> , they are represented by the same bar
<code>...</code>	graphical parameters

**Author(s)**

M. LAMBONI

graph.pc

*Principal Components graph for gsi objects***Description**

A function that plots the Principal components (PCs) and the sensitivity index on each PC

**Usage**

```
graph.pc(x, nb.plot = 15, nb.comp = NULL, xmax = NULL,
         beside = TRUE, ...)
```

**Arguments**

<code>x</code>	<code>gsi</code> object
<code>nb.plot</code>	number of input factors to be considered
<code>nb.comp</code>	number of PCs
<code>xmax</code>	a user-defined maximal $x$ value ( $x \leq 1$ ) in all the bar graphs that show sensitivity indices; or <code>NULL</code> if the user wants to keep default values
<code>beside</code>	if <code>TRUE</code> , the main and total sensitivity indices are represented by two bars; if <code>FALSE</code> , they are represented by the same bar
<code>...</code>	graphical parameters

**Author(s)**

M. LAMBONI

---

grpe.gsi	<i>Group factor GSI, obsolete function</i>
----------	--

---

**Description**

An obsolete function that computed the GSI of a group factor as one factor

**Usage**

```
grpe.gsi(GSI, fact.interet)
```

**Arguments**

GSI                      a gsi or dynsi object  
fact.interet    input factor to be grouped

**Note**

This is essentially an internal function for the multisensi package

**Author(s)**

M. LAMBONI

---

gsi	<i>Generalized Sensitivity Indices: GSI</i>
-----	---

---

**Description**

The gsi function implements the calculation of Generalised Sensitivity Indices. This method allows to compute a synthetic Sensitivity Index for the dynamic or multivariate models by using factorial designs and the MANOVA decomposition of inertia. It computes also the Sensitivity Indices on principal components

**Usage**

```
gsi(formula, model, factors, inertia = 0.95, normalized = TRUE,  
    cumul = FALSE, simulonly = FALSE, Name.File = NULL, ...)
```

**Arguments**

formula	ANOVA formula like "A+B+C+A:B" OR an integer equal to the maximum interaction order in the sensitivity model
model	output data.frame OR the name of the R-function which calculates the model output. The only argument of this function must be a vector containing the input factors values
factors	input data.frame (the design) if model is a data.frame OR a list of factors levels such as : <code>factor.example &lt;- list(A=c(0,1), B=c(0,1,4))</code>

<code>inertia</code>	cumulated proportion of inertia (a scalar $< 1$ ) to be explained by the selected Principal components OR number of PCs to be used (e.g 3)
<code>normalized</code>	logical value. TRUE (default) computes a normalized Principal Component analysis.
<code>cumul</code>	logical value. If TRUE the PCA will be done on the cumulative outputs
<code>simulonly</code>	logical value. If TRUE the program stops after calculating the design and the model outputs
<code>Name.File</code>	optional name of a R script file containing the R-function that calculates the simulation model. e.g <code>"exc.ssc"</code>
<code>...</code>	possible fixed parameters of the model function

### Details

If `factors` is a list of factors, the `gsi` function generates a complete factorial design. If it is a `data.frame`, `gsi` expects that each column is associated with an input factor.

### Value

`gsi` returns a list of class "gsi", containing all the input arguments detailed before, plus the following components:

<code>X</code>	a <code>data.frame</code> containing the experimental design (input samples)
<code>Y</code>	a <code>data.frame</code> containing the output matrix (response)
<code>SI</code>	a <code>data.frame</code> containing the Sensitivity Indices (SI) on PCs and the Generalized SI (GSI)
<code>mSI</code>	a <code>data.frame</code> of first order SI on PCs and first order GSI
<code>tSI</code>	a <code>data.frame</code> containing the total SI on PCs and the total GSI
<code>iSI</code>	a <code>data.frame</code> of interaction SI on PCs and interaction GSI
<code>cor</code>	a <code>data.frame</code> of correlation between PCs and outputs
<code>inertia</code>	vector of inertia per PCs and global criterion
<code>Rsquare</code>	vector of dynamic coefficient of determination

### Author(s)

M. Lamboni

### References

- M. Lamboni, D. Makowski and H. Monod, 2009. Multivariate global sensitivity analysis for dynamic crop models. *Field Crops Research*, volume 113. pp. 312-320
- M. Lamboni, D. Makowski and H. Monod, 2009. Multivariate sensitivity analysis to measure global contribution of input factors in dynamic models. Submitted to *Reliability Engineering and System Safety*.

### See Also

`dynsi`

**Examples**

```
##---- Should be DIRECTLY executable !! ----
# Test case : the Winter Wheat Dynamic Models (WWDM)
# input factors design
data(plan)
# input climate variable
data(Climat)
GSI <- gsi(2, biomasse, plan, inertia=3, normalized=TRUE, cumul=FALSE, climdata=Climat)
summary(GSI)
print(GSI)
plot(x=GSI, beside=FALSE)
#plot(GSI, nb.plot=4)           # the 'nb.plot' most influent factors
                                # are represented in the plots
#plot(GSI,nb.comp=2, xmax=1)    # nb.comp = number of principal components
#plot(GSI,nb.comp=3, graph=1)   # graph=1 for first figure; 2 for 2nd one
                                # and 3 for 3rd one; or 1:3 etc.
#graph.bar(GSI,col=1, beside=F) # sensitivity bar plot on the first PC
#graph.bar(GSI,col=2, xmax=1)   #
```

plan

*A factorial input design for the main example***Description**

Factorial design (resolution V) data for the 7 WWDM model input factors

**Usage**

```
data(plan)
```

**Format**

A data frame with 2187 observations on the following 7 variables.

Eb First WWDM input factor name  
 Eimax Second WWDM input factor name  
 K Third WWDM input factor name  
 Lmax Fourth WWDM input factor name  
 A Fifth WWDM input factor name  
 B Sixth WWDM input factor name  
 TI Seventh WWDM input factor name

**See Also**

[biomasse](#)

**Examples**

```
data(plan)
## maybe str(plan) ; plot(plan) ...
```

---

planfact	<i>Complete factorial design in lexical order</i>
----------	---

---

**Description**

Function that generates a complete factorial design in lexical order

**Usage**

```
planfact(nb.niv, make.factor = TRUE)
```

**Arguments**

nb.niv	vector containing the number of each input levels
make.factor	logical value. If TRUE the columns of the output are of class factor

**Value**

plan	data frame of the complete factorial design
------	---

**Note**

This is essentially an internal function for the multisensi package

**Author(s)**

M. LAMBONI

---

planfact.as	<i>Complete factorial design</i>
-------------	----------------------------------

---

**Description**

Computation of a complete factorial design for model input factors

**Usage**

```
planfact.as(input)
```

**Arguments**

input	list of factor levels
-------	-----------------------

**Value**

comp2	complete factorial design of model input
-------	--

**Note**

This is essentially an internal function for the multisensi package

**Author(s)**

M. LAMBONI

---

plot.dynsi	<i>Plot method for dynamic sensitivity results</i>
------------	--

---

**Description**

Plot method for dynamic sensitivity results of class dynsi

**Usage**

```
## S3 method for class 'dynsi':
plot(x, text.tuning = NULL, ...)
```

**Arguments**

x	a dynsi object
text.tuning	NULL or a small integer to improve the position of input factor labels
...	graphical parameters

**Details**

For labels that would be partly positioned outside the plot frame, the argument "text.tuning" may allow to get a better positioning. If it is equal to  $n$ , say, these labels are moved by  $n$  positions inside the frame, where 1 position corresponds to 1 output variable on the x-axis.

**Author(s)**

M. LAMBONI

**See Also**[dynsi](#)


---

plot.gsi	<i>Plot method for generalised sensitivity analysis</i>
----------	---

---

**Description**

Plot method for generalised sensitivity analysis of class gsi

**Usage**

```
## S3 method for class 'gsi':
plot(x, nb.plot = 10, nb.comp = 3, graph = 1:3, xmax=NULL,
      beside=TRUE, ...)
```

**Arguments**

<code>x</code>	a gsi object
<code>nb.plot</code>	number of input factors to be considered
<code>nb.comp</code>	number of Principal Components to be plotted
<code>graph</code>	figures number: 1 or 2 or 3
<code>xmax</code>	a user-defined maximal $x$ value ( $x \leq 1$ ) in all the bar graphs that show sensitivity indices; or NULL if the user wants to keep default values
<code>beside</code>	if TRUE, the main and total sensitivity indices are represented by two bars; if FALSE, they are represented by the same bar
<code>...</code>	graphical parameters

**Author(s)**

M. LAMBONI

**See Also**[gsi](#)

---

`print.dynsi`*print DYNsi*

---

**Description**

A function to print DYNsi results

**Usage**

```
## S3 method for class 'dynsi':
print(x, ...)
```

**Arguments**

<code>x</code>	a dynsi object
<code>...</code>	print parameters

**Author(s)**

M. LAMBONI

**See Also**[dynsi](#)



---

print.gsi	<i>print GSI</i>
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---

**Description**

function to print GSI results

**Usage**

```
## S3 method for class 'gsi':
print(x, ...)
```

**Arguments**

x	a gsi object
...	print parameters

**Author(s)**

M. LAMBONI

**See Also**

[gsi](#)

---

quality	<i>quality of any approximation</i>
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---

**Description**

Function that computes the sensitivity quality after making some assumptions about the number of PCs and the number of interactions

**Usage**

```
quality(echsimul, echsimul.app, normalise = TRUE)
```

**Arguments**

echsimul	model outputs
echsimul.app	Predicted model output
normalise	logical value

**Value**

A list with the following components:

**moy** mean

**biais** biais

**coef.det** R-square

**Note**

This is essentially an internal function for the multisensi package

**Author(s)**

M. LAMBONI

---

 simulmodel

*Model simulation*


---

**Description**

Function that simulates the model outputs

**Usage**

```
simulmodel(model, plan, nomFic = NULL, verbose = FALSE, ...)
```

**Arguments**

model	name of R-function
plan	data frame of input design
nomFic	name of file that contains the model function
verbose	verbose
...	... possible fixed parameters of the R-function

**Details**

The model function must be a R-functions. Models defined as functions will be called once with an expression of the form  $\mathbf{y} \leftarrow \mathbf{f}(\mathbf{X})$  where  $\mathbf{X}$  is a vector containing a combination of levels of the input factors, and  $\mathbf{y}$  is the output vector of length  $q$ , where  $q$  is the number of output variables

**Value**

data frame of model outputs

**Note**

This is essentially an internal function for the multisensi package

**Author(s)**

M. LAMBONI

---

summary.dynsi	<i>dynsi summary</i>
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---

**Description**

Function to summarize the dynamic sensitivity results

**Usage**

```
## S3 method for class 'dynsi':  
summary(object, ...)
```

**Arguments**

object	a dynsi object
...	summary parameters

**Author(s)**

M. LAMBONI

**See Also**

[dynsi](#)

---

summary.gsi	<i>summary of GSI results</i>
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---

**Description**

function to summarize the GSI results

**Usage**

```
## S3 method for class 'gsi':  
summary(object, ...)
```

**Arguments**

object	a GSI object
...	summary parameters

**Author(s)**

M. LAMBONI

**See Also**

[gsi](#)

yapprox

*Prediction based on PCA and anovas*

---

**Description**

A function that predicts the model output after PCA and aov analyses

**Usage**

```
yapprox(ACP, nbcomp = 2, aov.obj)
```

**Arguments**

ACP	prcomp object
nbcomp	number of PCs
aov.obj	aov object

**Value**

A list with components

**Y** model output predictions

**trace** model output inertia

**Note**

This is essentially an internal function for the multisensi package

**Author(s)**

M. LAMBONI

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